MNG

Radio Networks
Methods: Network Graphs

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Outline

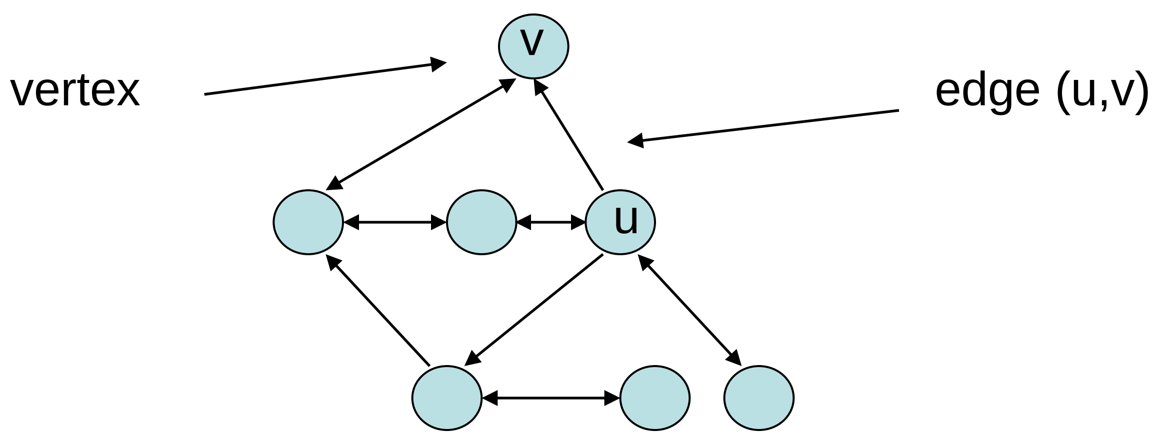
1. Introduction
2. Elements of Graph Theory
3. Communication Graph
4. Topology Control
5. Interference and Conflict Graphs
6. Dijkstra’s Algorithm

1. Introduction

Definition of Graph

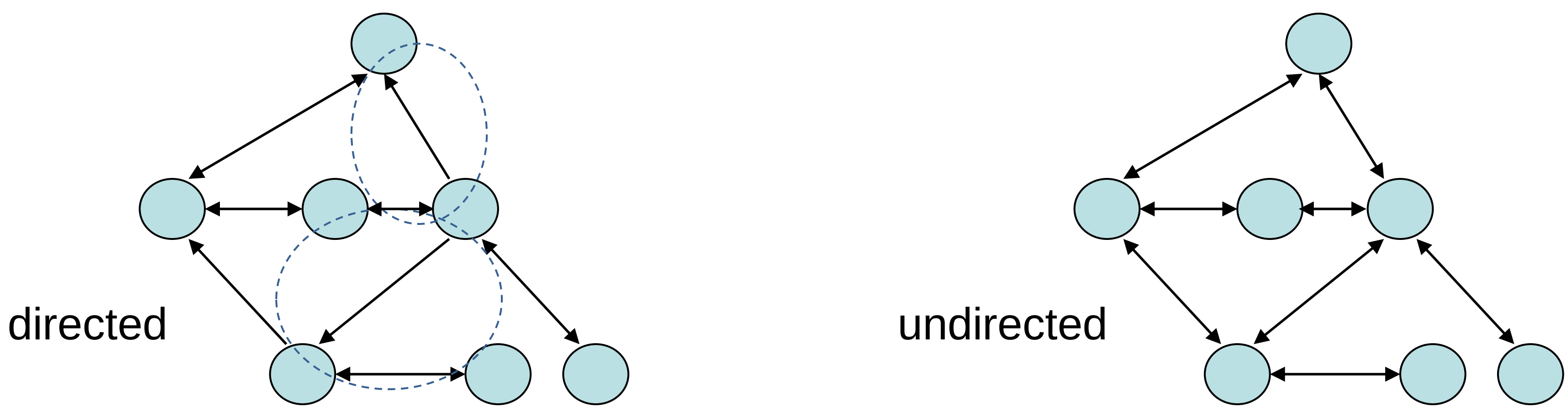
Graph: a set of items connected by *edges*. Each item is called a *vertex* or node. A graph can be seen as a *set* of vertices and a *binary relation* between them, named adjacency.

Mathematical Definition: A graph G can be defined as a pair (V,E), where V is a set of vertices, and E is a set of edges between the vertices: E = {(u,v) | u, v in V}, where (u,v) is an ordered pair.



Directed and Undirected Graphs

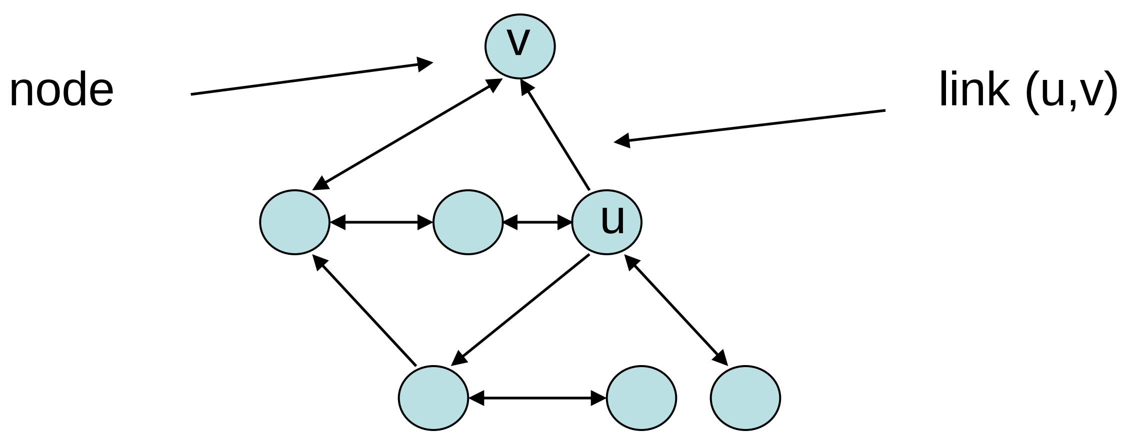
If the graph is **undirected**, the adjacency relation defined by the edges is *symmetric*, i.e. E = {{u,v} | u, v in V} (sets of vertices rather than ordered pairs).



A Network in Graph Theory

A Network is a pair (N,P).

N is a set of nodes, of size n. P is the function mapping every node u to a position P(u). If a rule provides existence conditions for edges among nodes, a graph is obtained where the vertices are nodes, and the edges are links.



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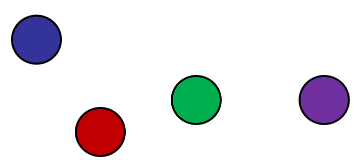
A Range Assignment is a function assigning to every node u a transmit range, RA(u). Rule: if the edge between u and an other node v is shorter than RA(u), then the edge (u,v) exists.



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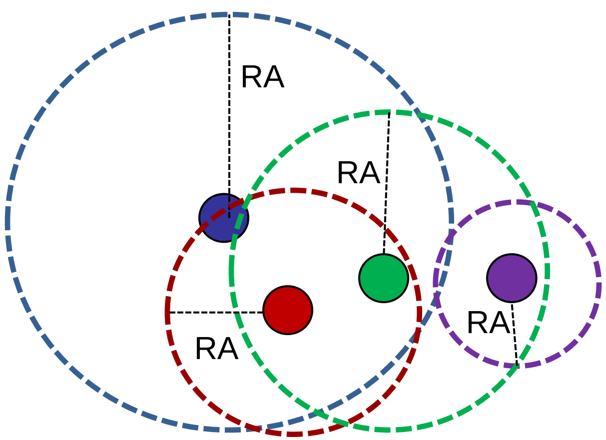

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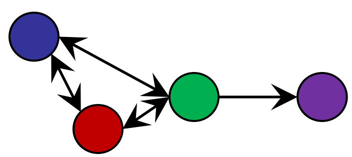

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A Range Assignment is a function assigning to every node u a transmit range, RA(u). Rule: if the edge between u and an other node v is shorter than RA(u), then the edge (u,v) exists.

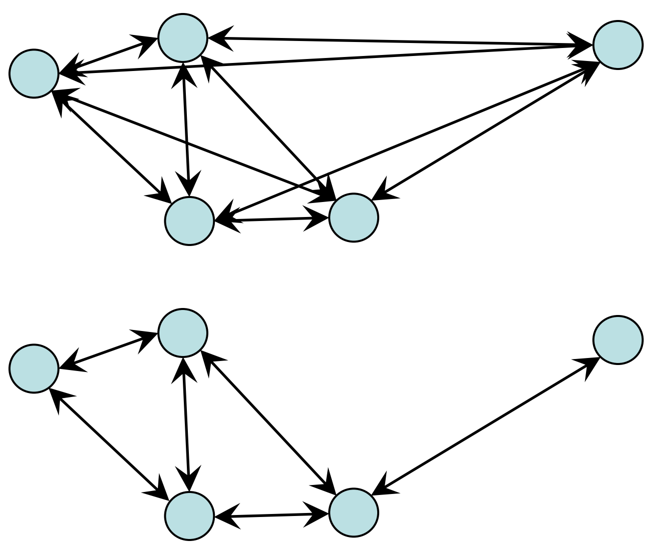
A Power Assignment is a function assigning to every node u a transmit power, PA(u). Rule: if the received power at an other node v is above the receiver sensitivity of v, then the edge (u,v) exists.

The latter case is closer to actual radio resource control in networks.

My Network is compact or sparse?

Graphs can help visualising whether a network is compact or sparse.

Therefore, they are useful to predict (and avoid) interferences.



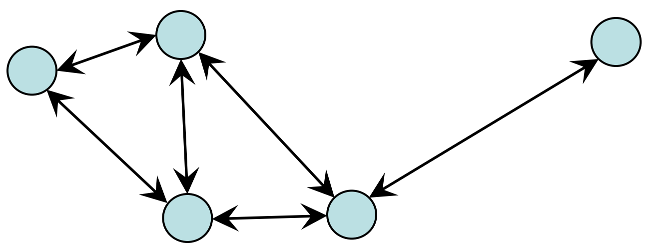
Topology Control

The **topology** of a network is the set of links *maintained* among nodes.

Topology Control is a protocol entity aiming at controlling the set of links, in order to simplify / permit flow of messages between nodes.

The ***physical* topology** of a network can be controlled through physical layer, in most cases power control techniques are used.

The ***logical* topology** of a network is controlled by entities working at layer 3, and is based on a **reduced set of links** wrt the physical topology.



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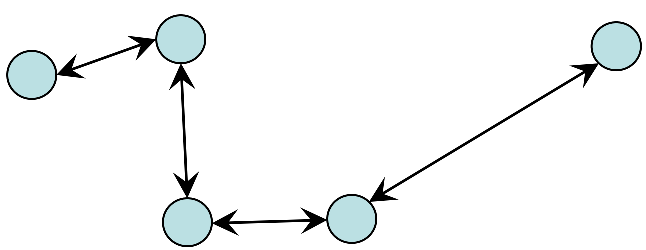


Inquiry Based Session

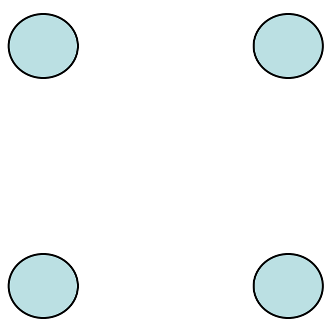
1. What is the maximum number M of edges in a Graph with n vertices?

2. Are Network Graphs based on PA normally directed or undirected?

3. What is the minimum number K of edges in an undirected Network where every node can reach any other, through a whatever large number of hops?

4. Compare bus, ring, star topologies under the viewpoint of the number of edges and the average number of hops to reach one node to an other.

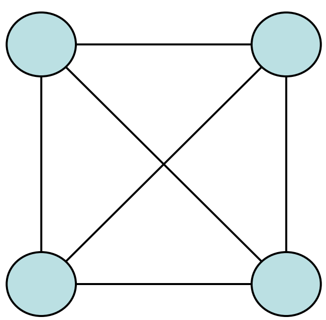
n = 4



Inquiry Based Session

1. What is the maximum number M of edges in a Graph with n vertices?

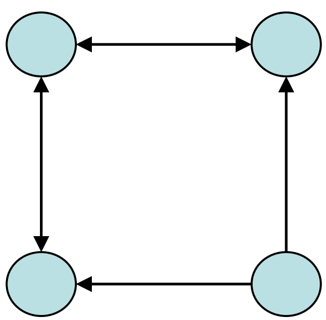
M = n (n-1) / 2

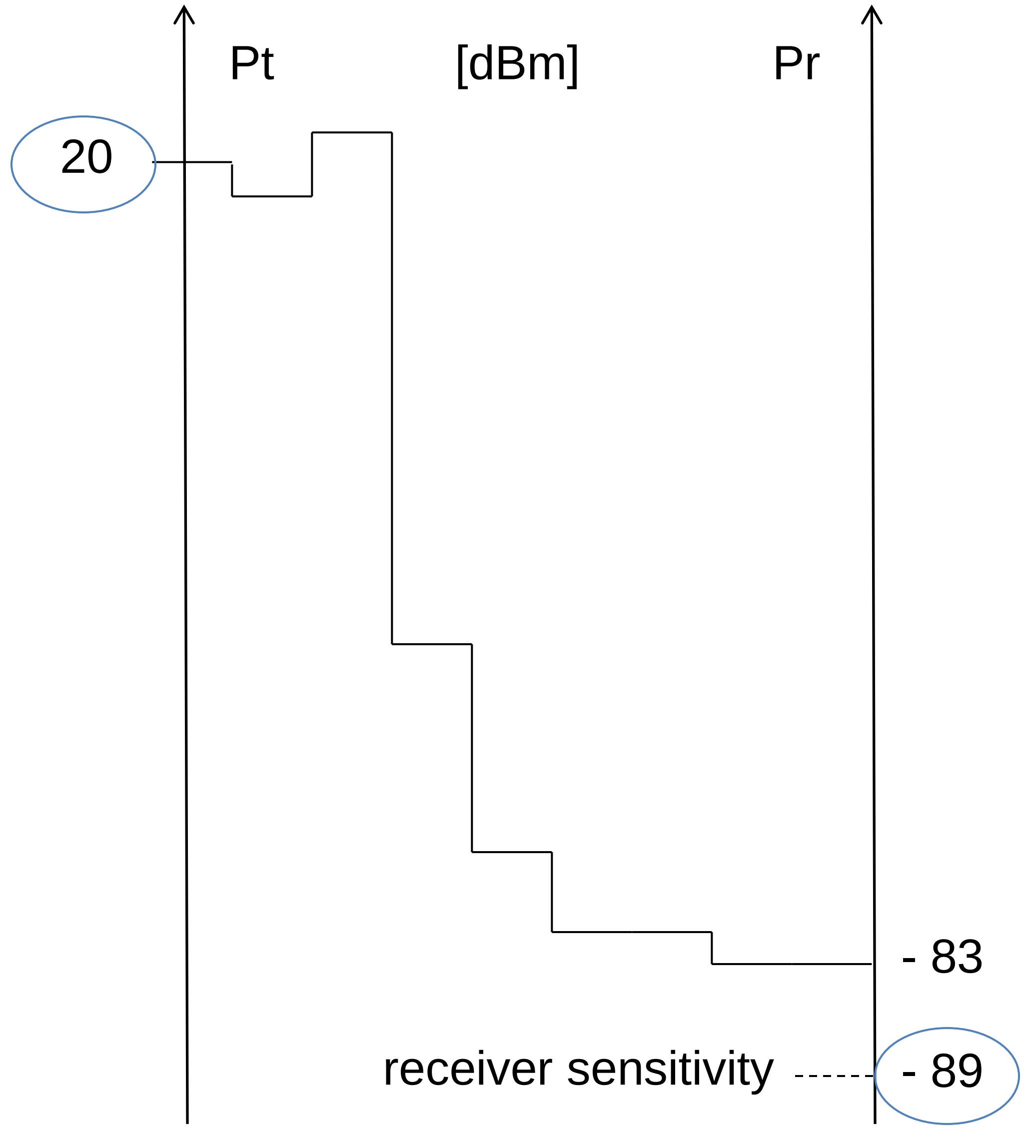


Inquiry Based Session

2. Are Network Graphs based on PA normally directed or undirected?

Pt		**20 dBm**
At	**-**	**2 dB**
Gt	**+**	**4 dB**
Aim	**-**	**80 dB**
Fading	**-**	**20 dB**
Shadowing	**-**	**4 dB**
Gr	**+**	**0 dB**
Ar	**-**	**1 dB**
Pr	**-**	**83 dBm**

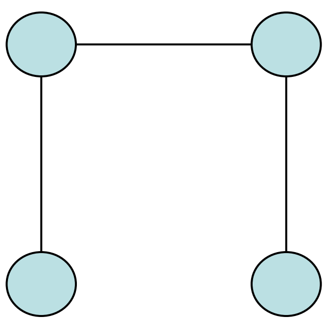




Inquiry Based Session

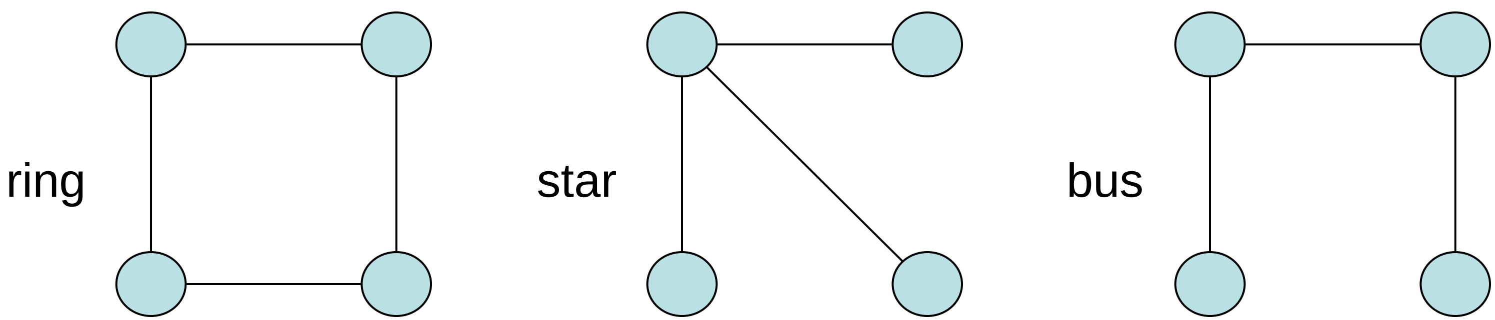
3. What is the minimum number K of edges in an undirected Network where every node can reach any other, through a whatever large number of hops?

K = (n-1)



Inquiry Based Session

4. Compare bus, ring, star topologies under the viewpoint of the number of edges and the average number of hops to reach one node to an other.



2. Elements of Graph Theory

Elements of Graph Theory

Geometric Graph (GG)

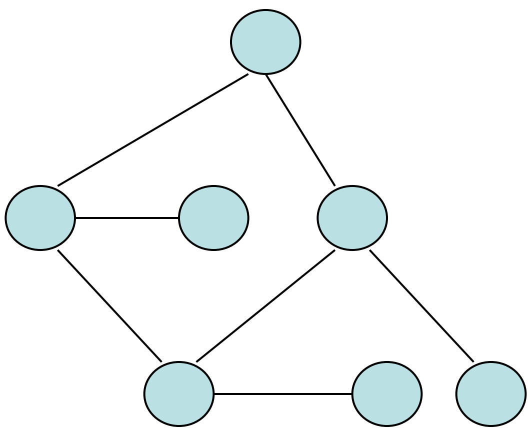
Vertices have a geometric location in $\mathbf{R}^d$. In the following we assume d = 2.

Random Graph

Edges between pairs of nodes exist according to random statistics.

Geometric Random Graph (GRG)

Random Graph where edges exist according to proximity relation between nodes and nodes are in unknown positions.

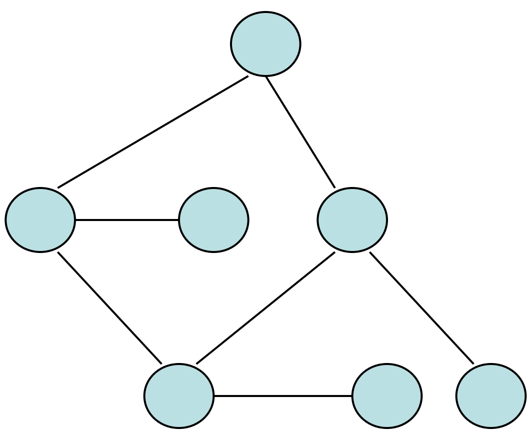


Elements of Graph Theory

Connected Graph
An undirected graph that has a path between every pair of vertices.

Edge Connectivity
The smallest number of edges whose deletion will cause a connected graph to not be connected.

Node Connectivity
The smallest number of vertices whose deletion causes a connected graph to not be connected.

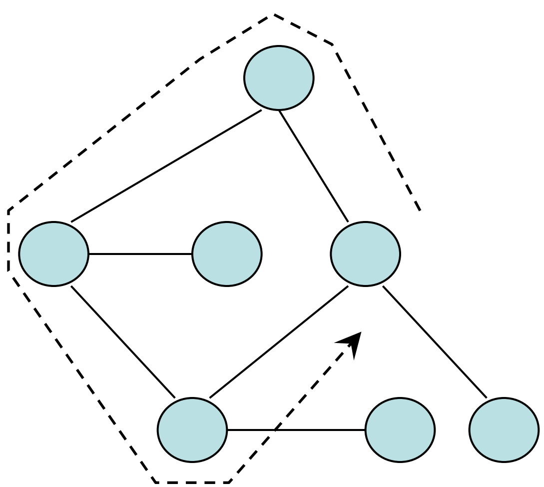


Connected
EC=1
NC=1

Elements of Graph Theory

Complete Graph
An undirected graph with an edge between every pair of vertices

Acyclic Graph
A graph with no *path* that starts and ends at the same *vertex*.



Not complete
Not acyclic

Elements of Graph Theory

Complete Graph
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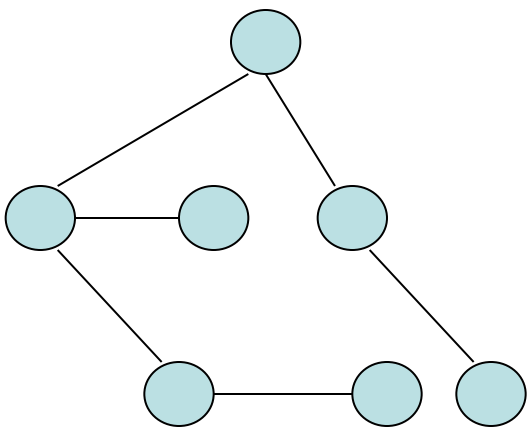


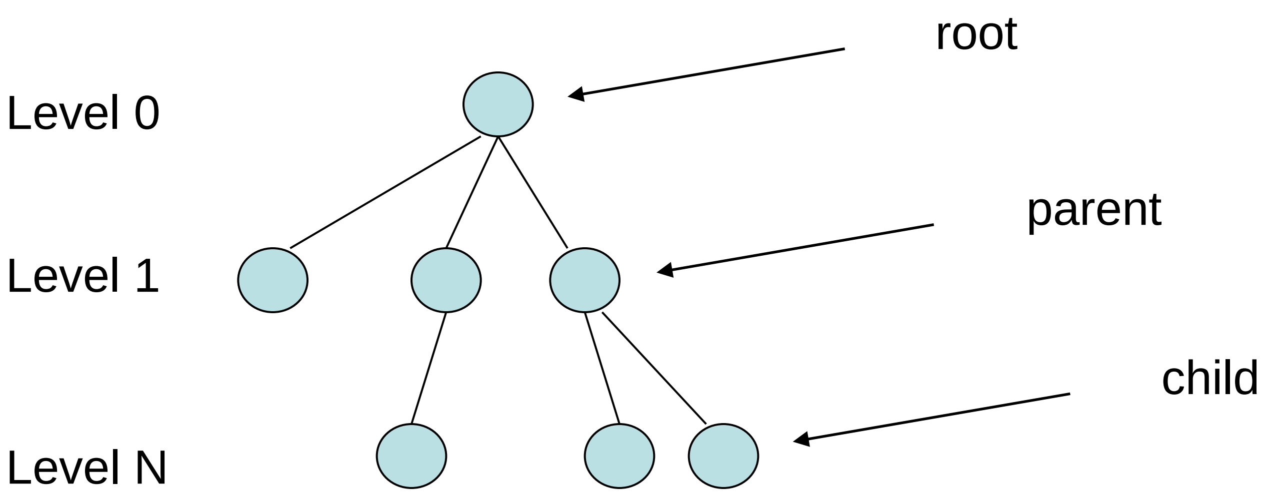
Not complete
Acyclic

Elements of Graph Theory

Tree
A connected, undirected, acyclic graph.

It is a data structure accessed beginning at the root node, where each node is either a leaf or an internal node. An internal node has one or more child nodes and is called the parent of its child nodes. All children of the same node are siblings.

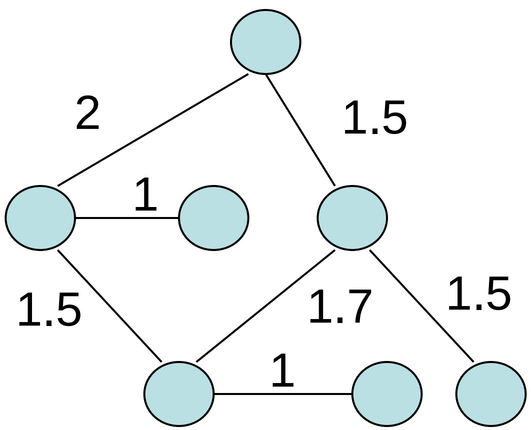
A simple tree composed of the root and only level 1 nodes, is named **Star**.



Elements of Graph Theory

Weighted Graph
A graph having a weight, or number, associated with each edge

Euclidean Tree
A tree in a weighted GG where weights are assigned to edges based on Euclidean distances.



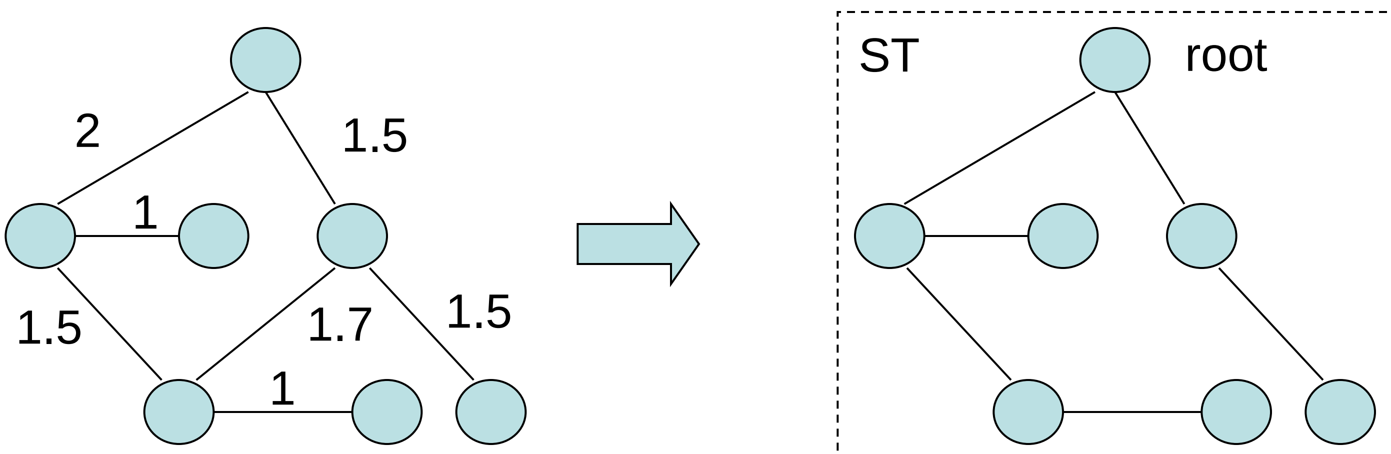
Elements of Graph Theory

Spanning Tree

A connected, acyclic undirected sub-graph containing all the vertices of a graph

Minimum Spanning Tree (MST)

A minimum-weight tree in a weighted graph which contains all of the graph's vertices.



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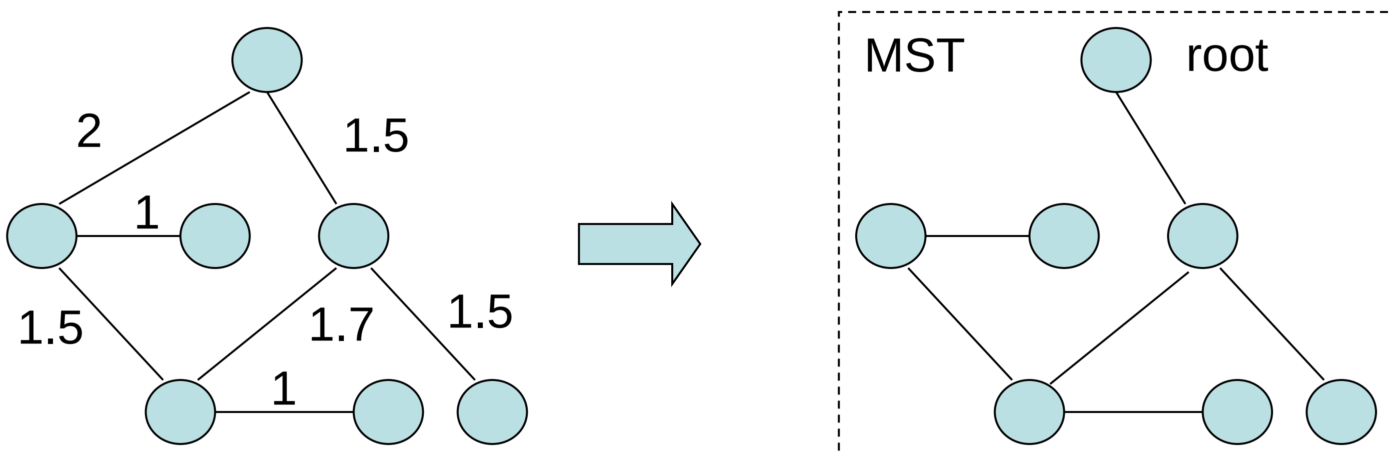


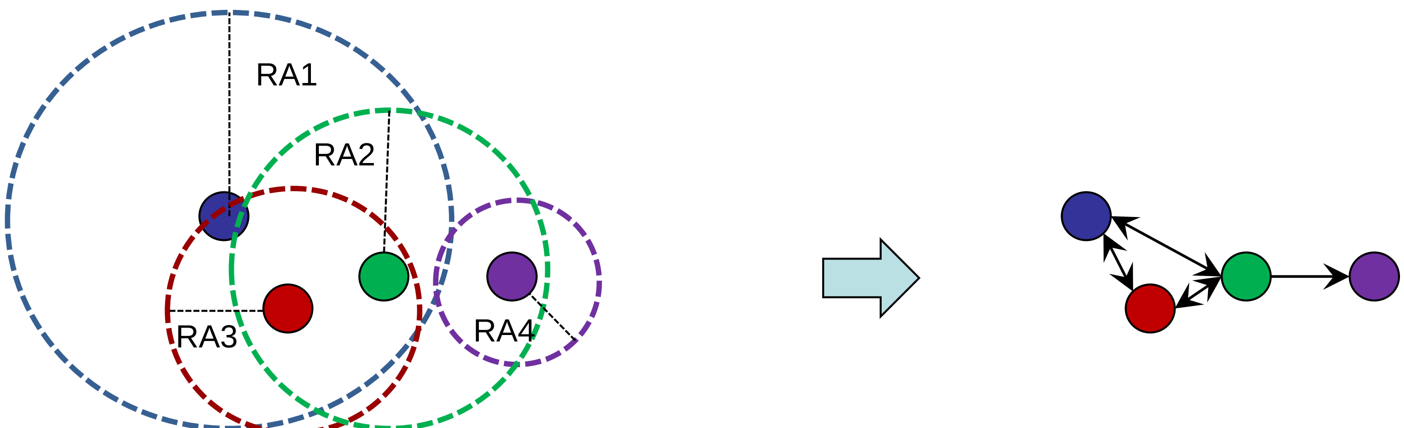
3. Communication Graphs

The Communication Graph

The Geometric Communication Graph is the directed graph (G,E) where the directed edge (u,v) exists if the Euclidean distance between u and v is less or equal than RA(u). In this case v is neighbour to u. If u is also neighbour to v for all pairs {u,v}, the Communication Graph is undirected and all links are symmetrical.

A RA for a Network is **connecting** if the correspondent GCG is connected.

A RA where all nodes have the same transmit range is said **homogeneous**.



not connecting

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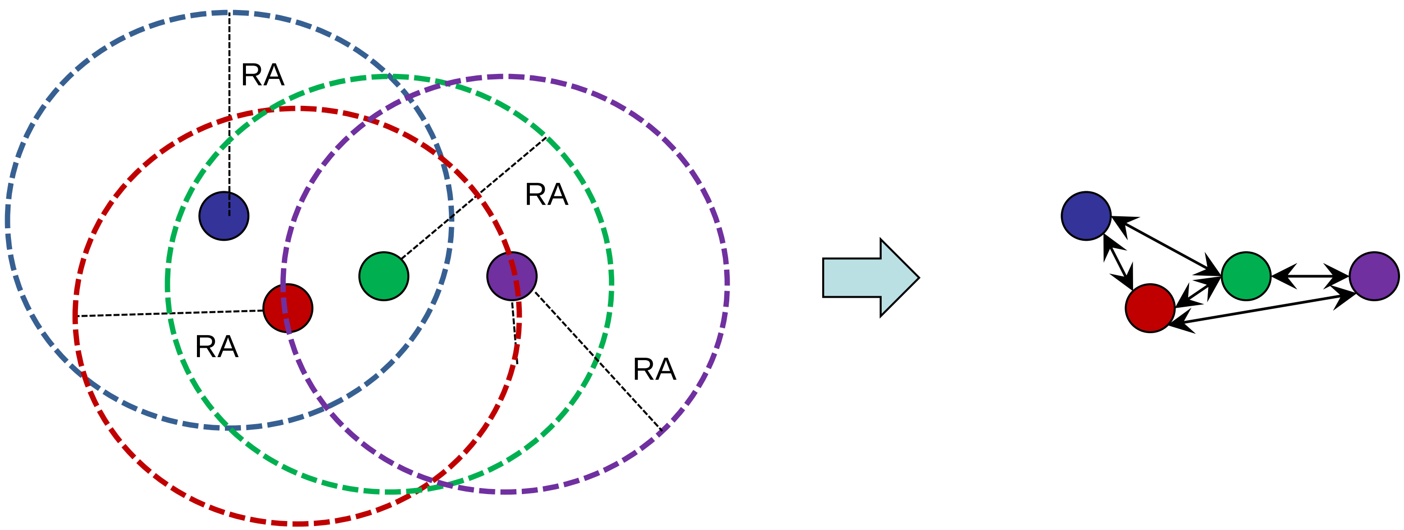


connecting
homogeneous

The Communication Graph

The Physical Communication Graph is the directed graph (G,E) where the directed edge (s,d) exists if the received power at d when only s is transmitting, is larger than the receiver sensitivity. In this case s is neighbour to d. If d is also neighbour to s for all pairs {s,d}, the Physical Communication Graph is undirected and all links are symmetrical.

A level of transmit power for a Network is **connecting** if the correspondent PCG is connected.

A PA where all nodes have the same transmit power is said **homogeneous**.

The Logical Communication Graph is a sub-graph of the PCG that includes all nodes and a subset of edges, selected according to any criteria.

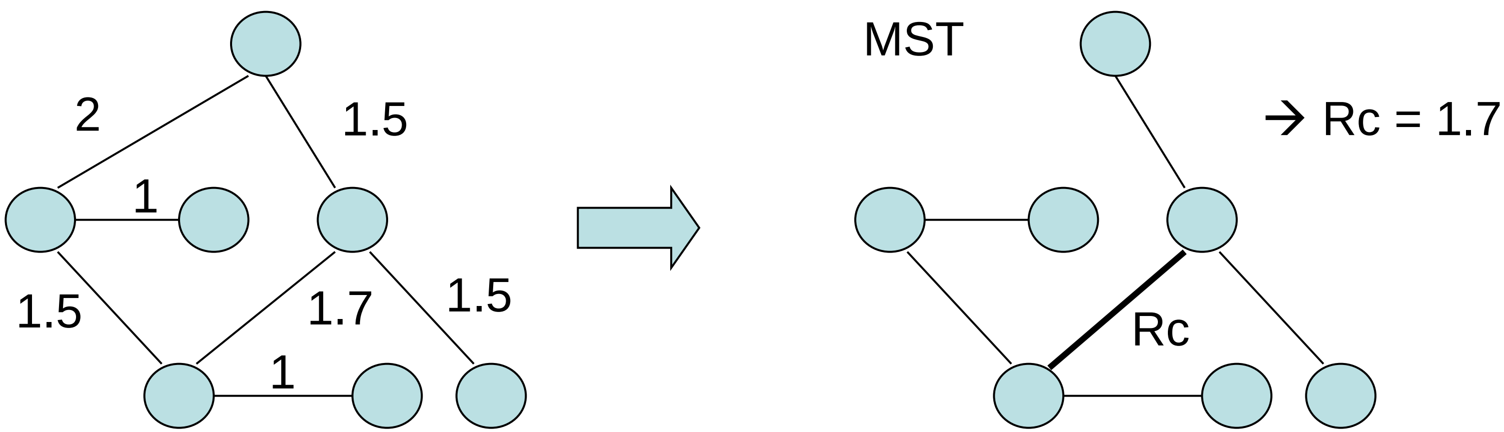
Depending on the received power definition, Communication Graphs can be Random Graphs or not (if short or long term average received power, or only median distance dependent power are considered respectively; in the latter case, the Communication Graph is also Geometric).

4. Topology Control

Critical Transmission Range

Critical Transmission Range:
The minimum value of R (with homogeneous RA), Rc, s. t. the LCG is connected.

Under a centralised approach, knowledge of the CTR would allow minimising topology complexity, and energy waste

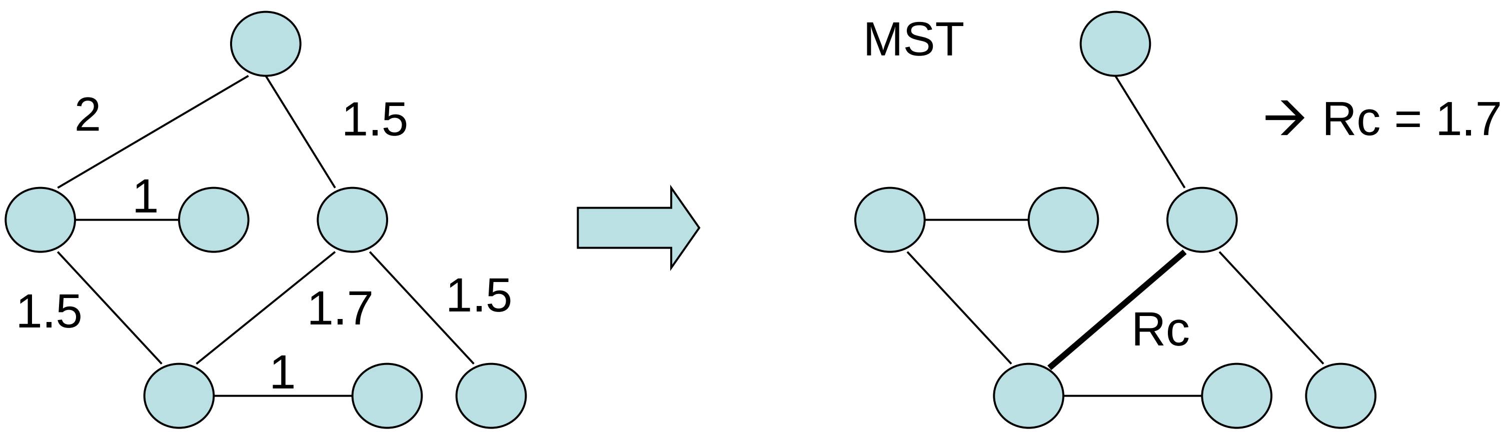


Critical Transmission Range

If the nodes in the Network are located in **known positions**:

the CTR for connectivity Rc of a Network equals the length of the longest edge of the Euclidean MST of the corresponding Physical Communication Graph

Assuming that only distance dependent loss is considered, from the CTR we can compute the minimum transmit power.

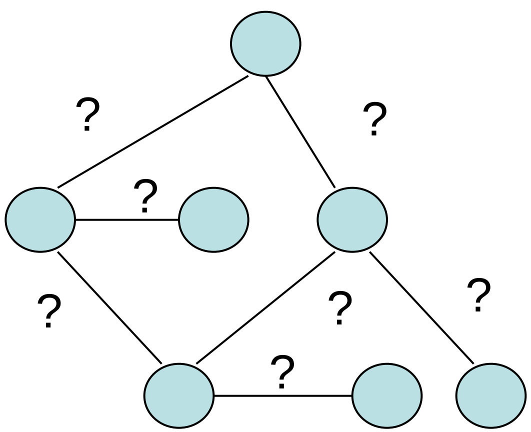


Critical Transmission Range

If the nodes in the Network are located in **unknown positions**:

the CTR for connectivity Rc of a Network is a random variable

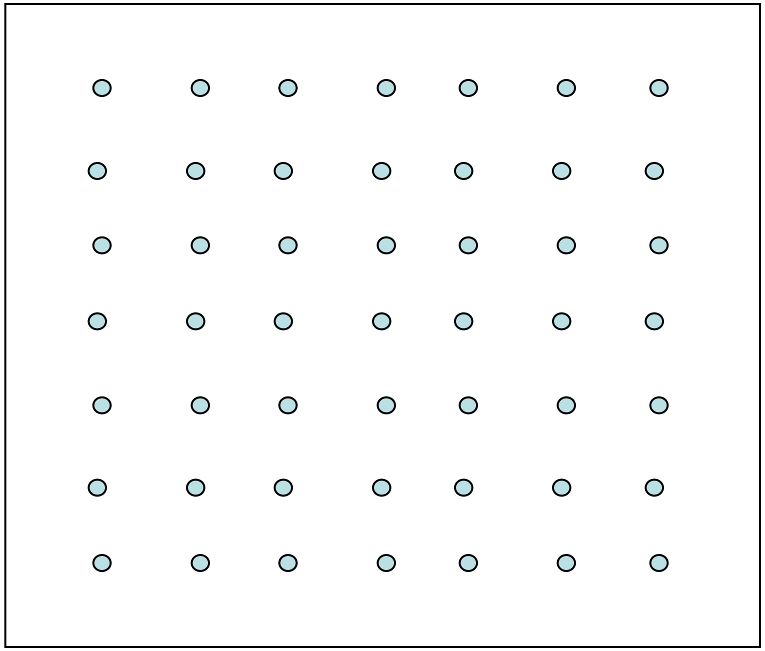
How to estimate it?
How does it scale with node density?



Critical Transmission Range

How does it scale with node density?

Unit square



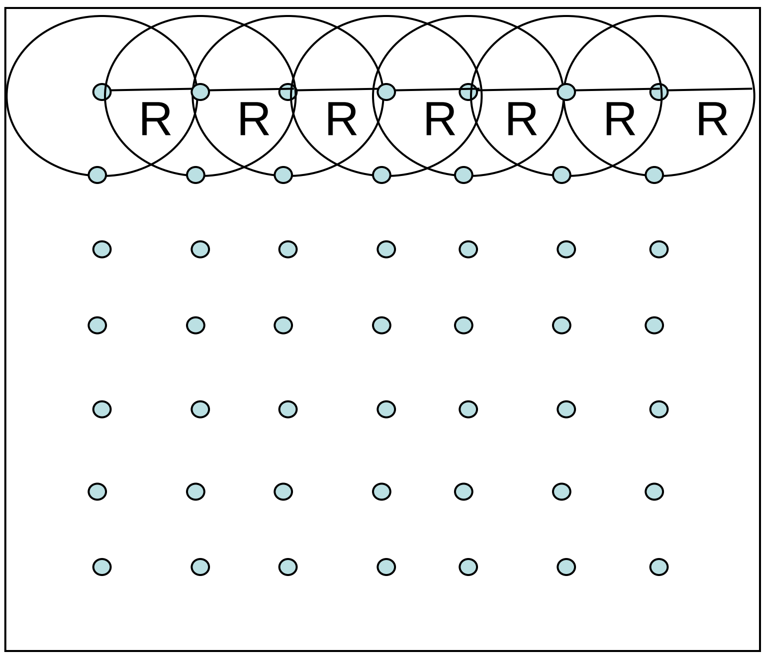
Regular grid as reference

Example: n = 49

Node density $\rho = n / 1 = n$

Critical Transmission Range

Unit square



CTR:

R s.t. R [sqrt(n) + 1] = 1

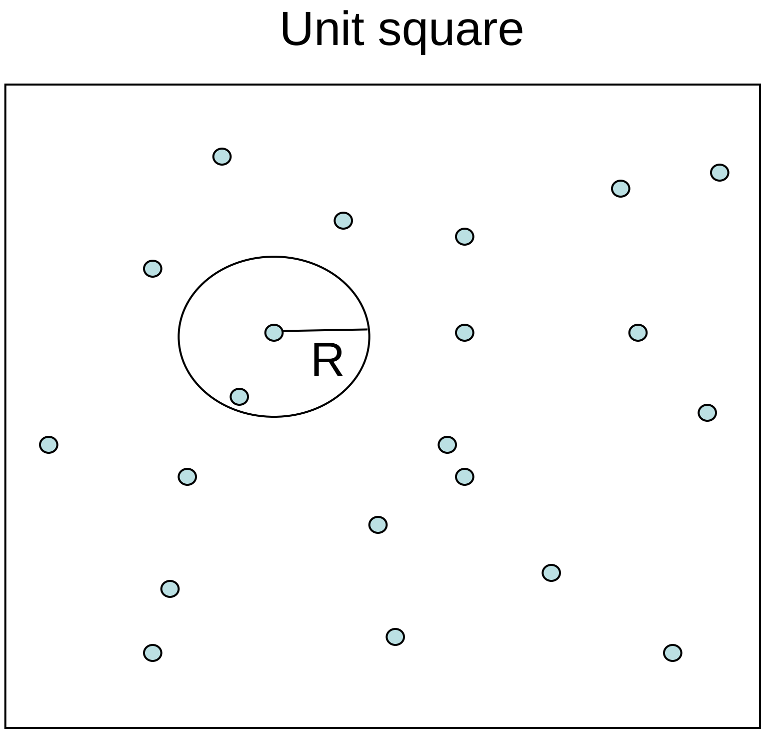
→ CTR = 1 / [sqrt(n) + 1]

In a square of side L

→ **CTR = L / [sqrt(n) + 1]**

Critical Transmission Range

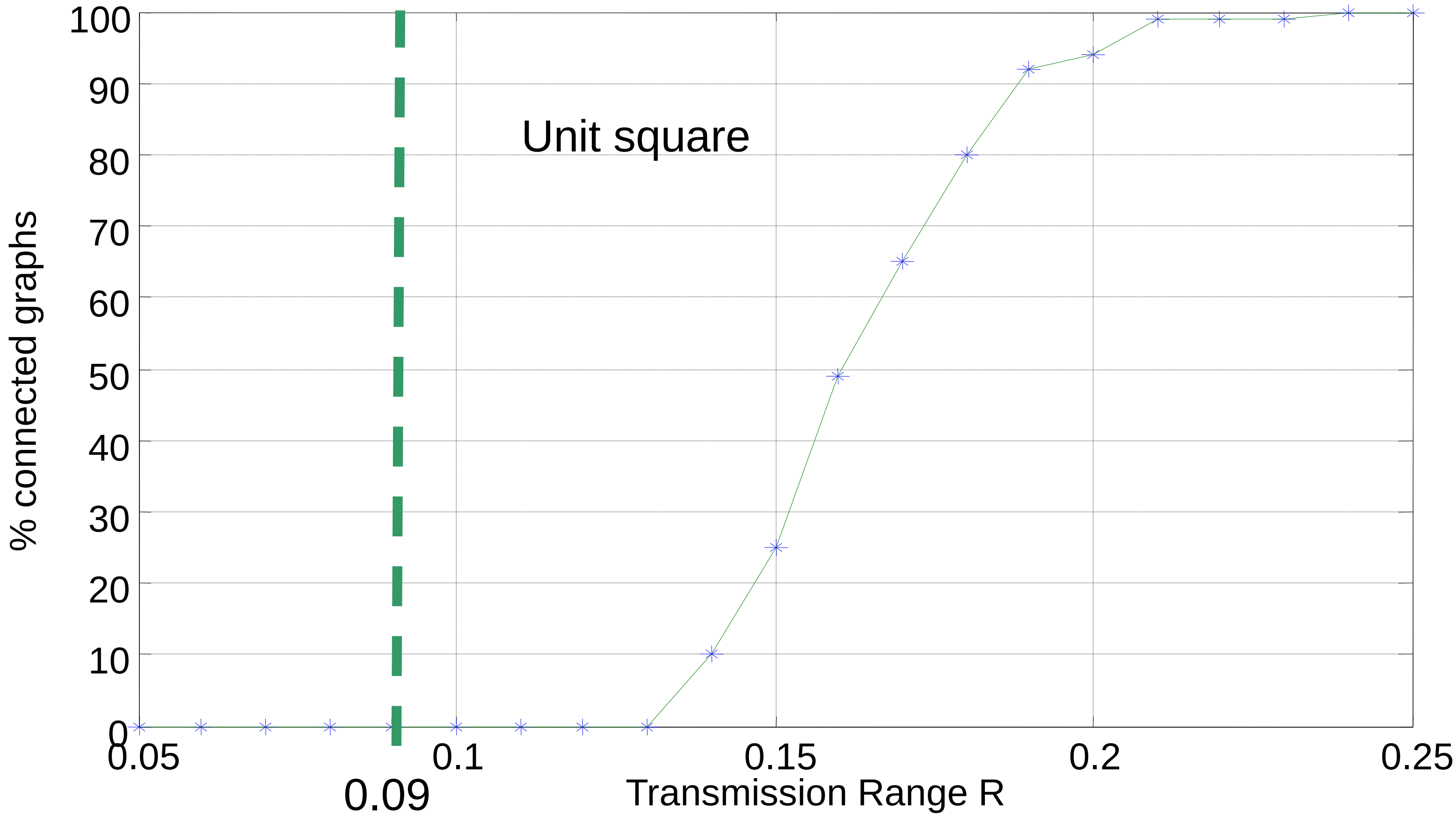
If the nodes in the Network are randomly distributed,
the CTR is a random variable.



Critical Transmission Range

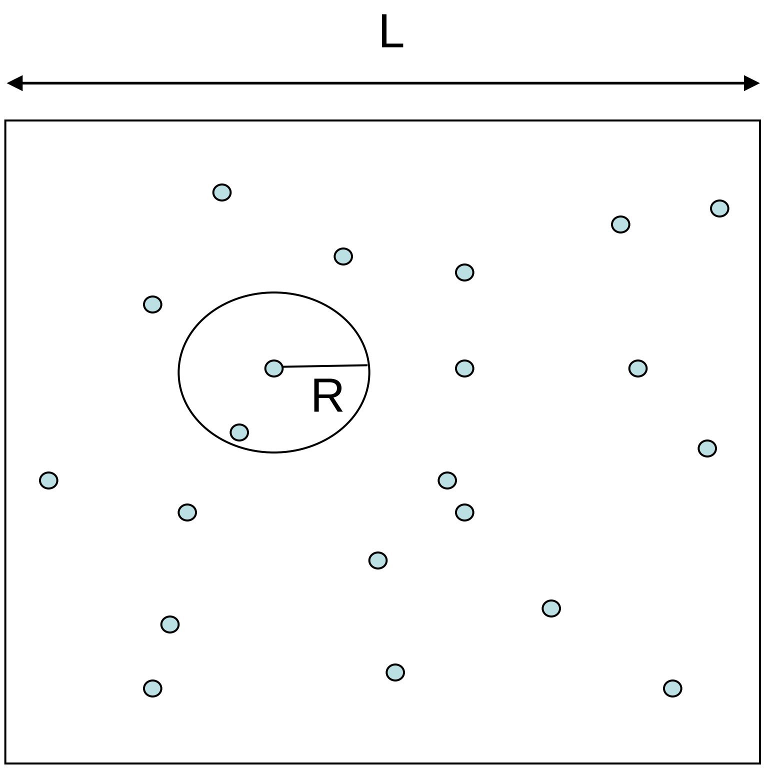
n = 100 → CTR = 0.09 in a regular grid

100 graphs



Randomness of node position has a significant impact on connectivity issues

Critical Transmission Range



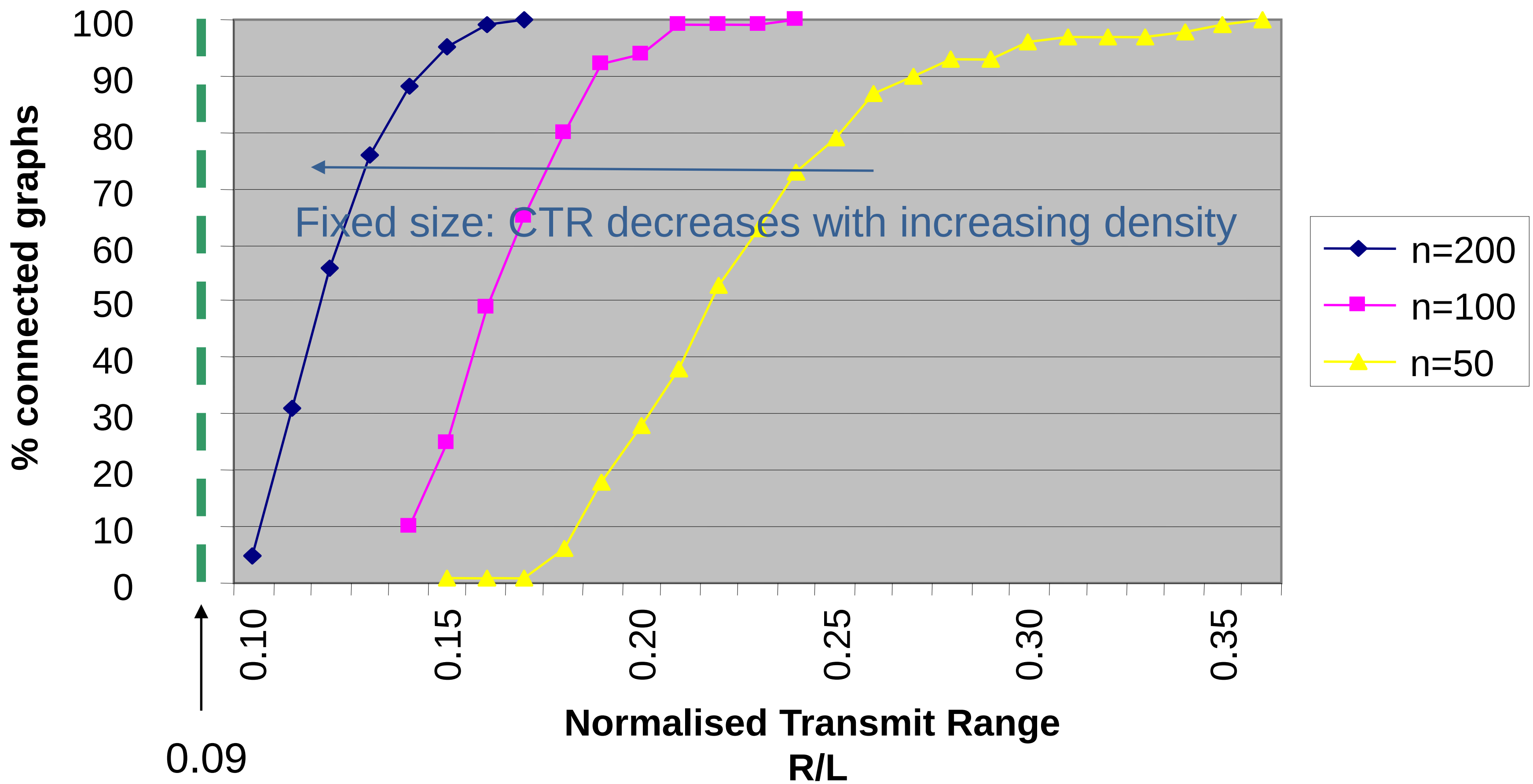
n number of nodes

→ node density ρ is n / L^2

→ $n = \rho L^2$

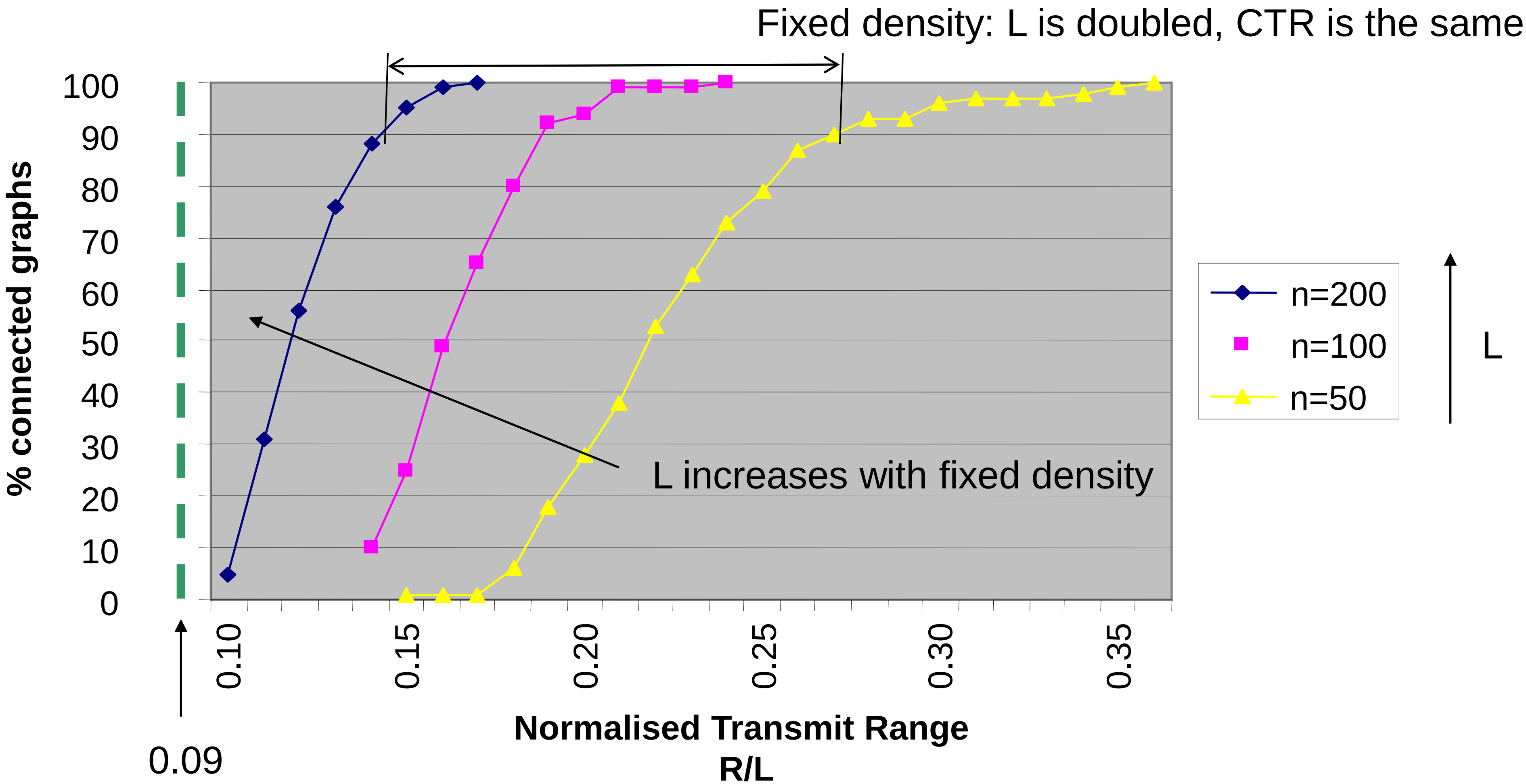
R transmission range

Critical Transmission Range



CTR seems to scale approximately as 1/sqrt(n)

Critical Transmission Range



Size of scenario has no significant impact on connectivity issues

Critical Transmission Range

Theorem (Penrose 1997):

Given the unit square and n nodes distributed randomly and uniformly, then the limit for n tending to infinity of
Prob [$n\pi (Rc)^2 - \log(n) \leq b$]
is
$1 / \exp(\exp(-b))$
for any b in $\mathbb{R}$ where Rc is the CTR.

As a corollary, for n tending to infinity we have (choose b tending to infinity)
Prob [$Rc \leq \text{sqrt}[(b + \log(n)) / n\pi)]$] = 1.

In other words, **sqrt [(b + log(n)) / nπ)] is an upper bound to Rc for n tending to infinity.**

For finite values of n, this expression does not necessarily represent an upper bound.

Critical Transmission Range

For which values of n are the predicted values of Rc accurate?

For instance, with b = log(log(n))

n	Rc (Penrose)	Rc (sim., 99% conf)	Rc=1 / [sqrt(n) + 1]
10	0.32	**0.66**	0.24
100	0.14	**0.23**	0.09
1000	0.05	**0.08**	0.03

Not very accurate for practical values of n.
It is accurate for very high values of node density.

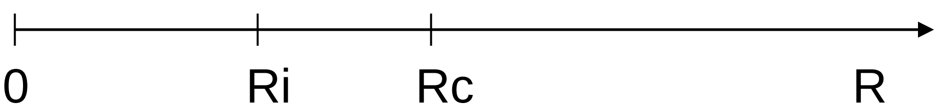
Considerations for n tending to infinite are often unrealistic for practical values of node densities.

The Giant Component

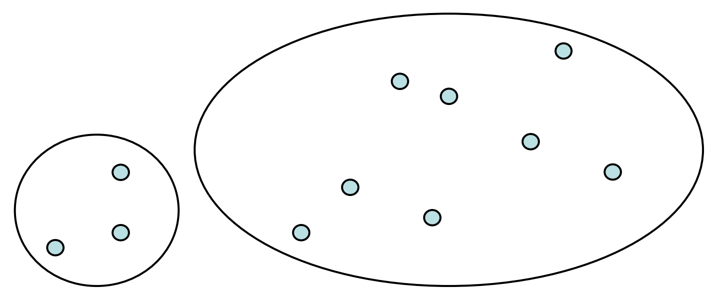
Consider a given graph, and let R increase starting from 0.

For R = 0 the graph is not connected.
When R increases, nodes group together in clusters.
When R = Ri the last isolated node disappears.
When R = Rc the graph becomes connected.

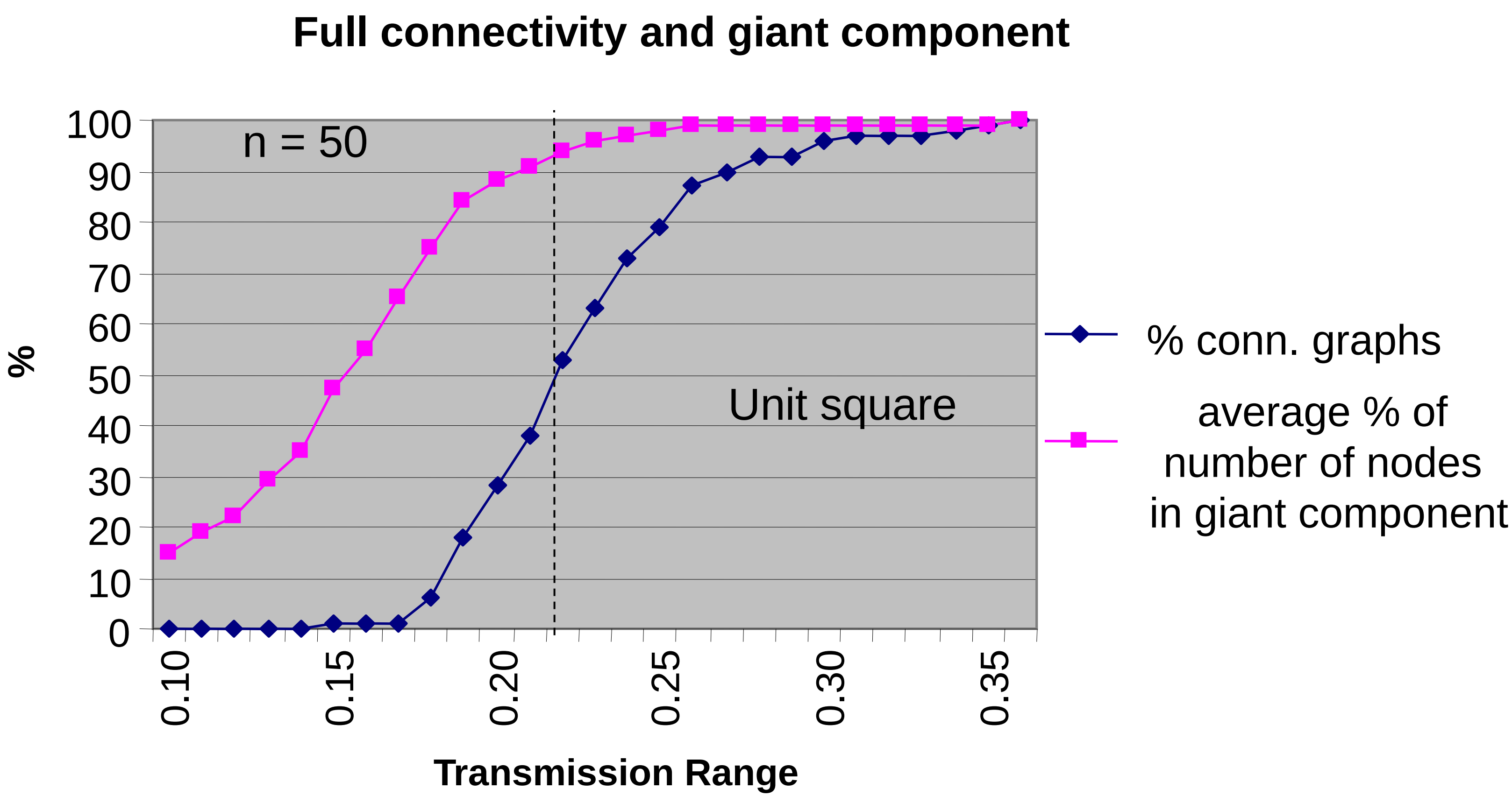
Clearly, Ri < Rc as for some Ri < RA < Rc there might be a communication graph not connected even in the absence of isolated nodes (a “clustered” network with isolated clusters).



For high densities, Ri ≈ Rc .



The Giant Component



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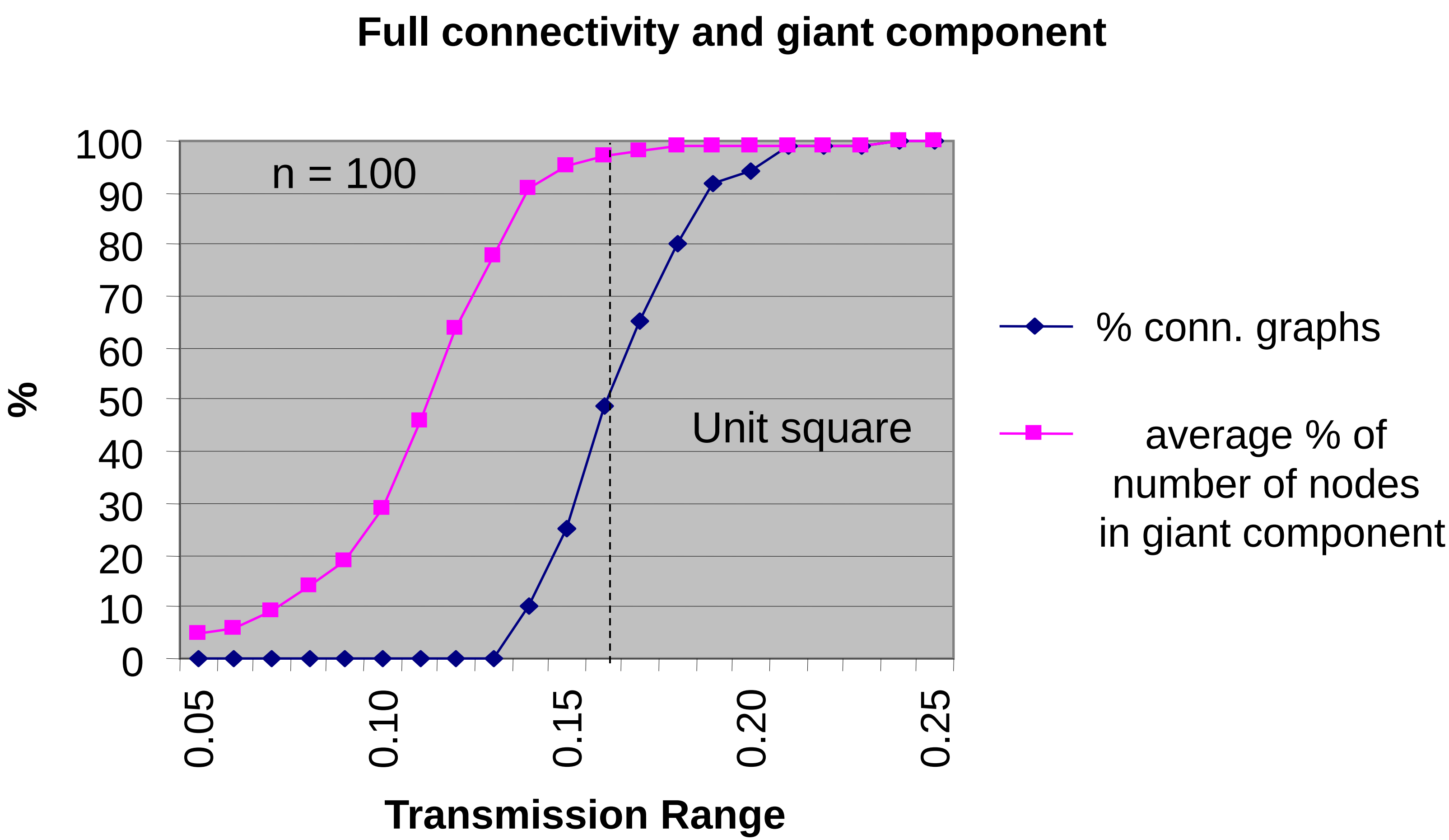

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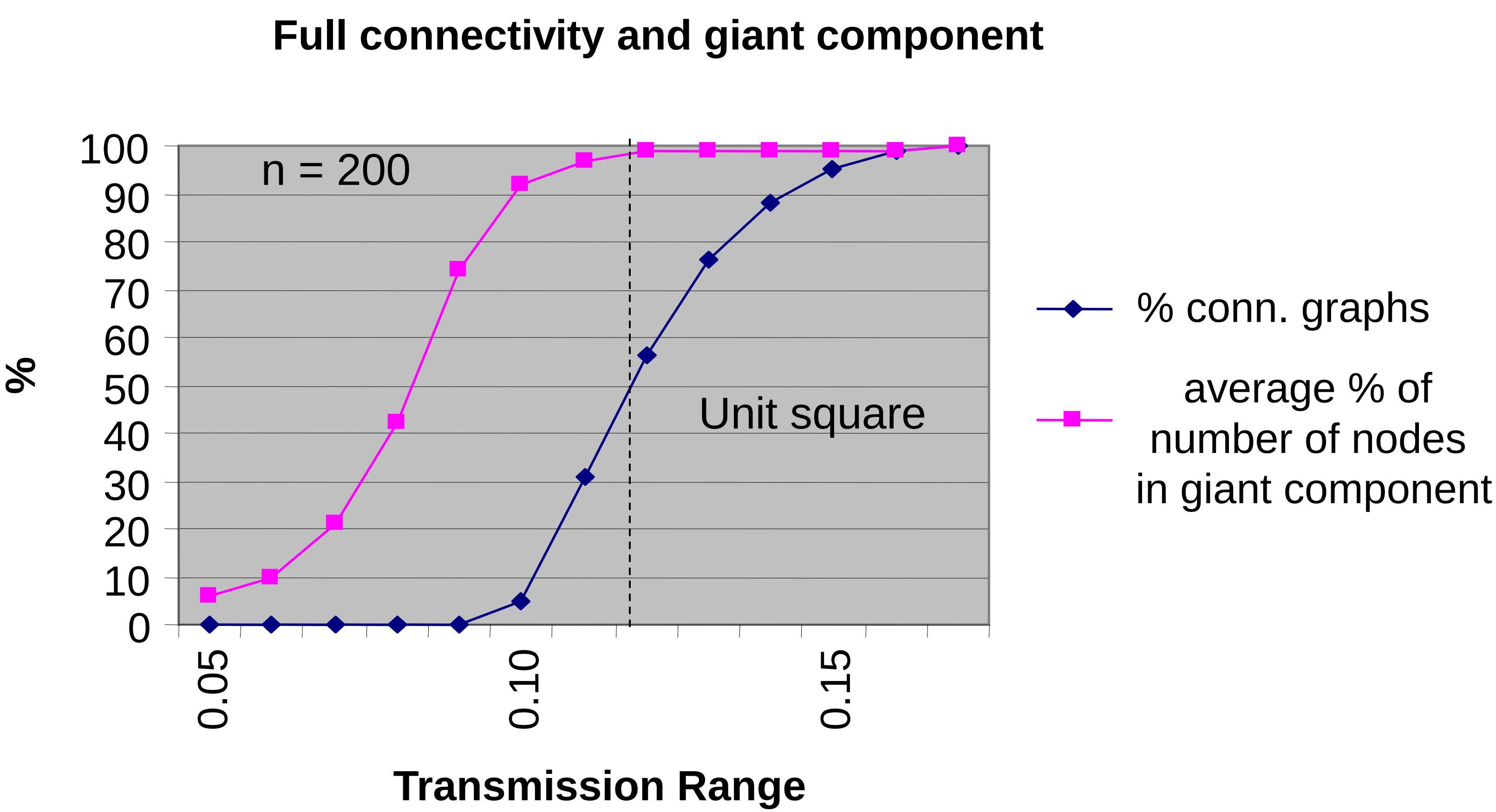


Critical Transmission Range

It can be claimed that, approximately, for any value of n

the probability of a Communication Graph to be connected equals the probability of no isolated nodes.

Computing the probability of isolated nodes as a function of Rc, the CTR can be estimated.

Assume an infinite PPP: number of nodes (uniformly distributed with density ρ) in any area of size A is Poisson, with mean equal to ρ * A

A node is isolated if there are zero nodes within its transmission range Rc. This happens with probability exp(- ρ * πRc^2).

So, the probability that a network is connected is roughly P =1 - exp(- ρ * πRc^2).

Rc is approximately equal to sqrt [- ln(1 – P) / (ρ * π)] proportional to 1 / sqrt (n)

Critical Transmission Range

Rc = sqrt [- ln(1 – P) / (ρ * π)]

For instance, with P = 0,99 → **Rc = sqrt [1.47 / ρ]**

n	Rc (Penrose)	Rc (sim., 99% conf)	Rc= sqrt (1,47 / n)
10	0.32	**0.66**	0.38
100	0.14	**0.23**	0.12
1000	0.05	**0.08**	0.04

Not very accurate too.

Simple derivation of the CTR remains an unsolved issue.

5. Interference and Conflict Graphs

The Interference Graph

The Interference Graph is the directed graph (G,E) where the directed edge (i,r) exists if the received power at r when only i is transmitting, is larger than the level of power causing a packet loss at r, when any other node included in the Logical Communication Graph is transmitting (alone).

The Conflict Graph

The Conflict Graph is the directed graph (G,E) where the directed edge (j,k) exists if the transmission over link j causes a packet loss at the link k, when no other link is active. Its vertices are the links in the Logical Communication Graph.

The Conflict Graph can be used to schedule transmissions in a network, when links are not implementing Adaptive Modulation and Coding.

Exercise MNG#1

A network is composed of eight nodes located on the perimeter of a square of side 100m: four on the edges, and four on the median points of the four sides. The received power is equal to $kd^{-\beta}$ mW, where k = 0.001 when transmit power is set at 0 dBm (default). The propagation exponent β is equal to 3. The receiver sensitivity for all nodes is – 95 dBm.
Draw the PCG of the network.

Assume each node will maintain only the four strongest links.
Draw the LCG of the network.

Assume the protection ratio is 1.5 dB.
Draw the interference and conflict graphs.

Compute the CTR and the minimum level of transmit power needed to keep the network connected.

6. Dijkstra’s Algorithm

Dijkstra: Scope

Given a weighted LCG (where weights are non negative numbers), the Dijkstra's algorithm finds the shortest path (defined as the path with minimum sum of weights) between a source and destination node. For this reason, it is considered an optimum routing algorithm for a mesh network.

However, it requires knowledge of all weights in the graph.

Actually, it finds the shortest path from the source to all other nodes. Therefore, it finds the MST conditioned to the root in the given source (the actual MST of the LCG might be different, having a different root).

If weights are not Euclidian and account for the reliability of links, the algorithm provides the paths having largest reliability.

The algorithm can be also used to define the LCG from a PCG, if the root of the tree is defined a priori.

Dijkstra: Algorithm

Denote as N the number of nodes in the LCG, including the source.
The algorithm is iterative, with N-1 iterations.

Denote as U_0 the source node and as V_0 the destination (irrelevant).
Denote as S_0 the set containing U_0.
Denote as S_i the set containing, besides the source, all nodes for which the shortest path from the source has been found after the i-th iteration.
Denote as U_i the node included in the set S_i at the i-th iteration.
Denote as S_i^c the set complementary to S_i.

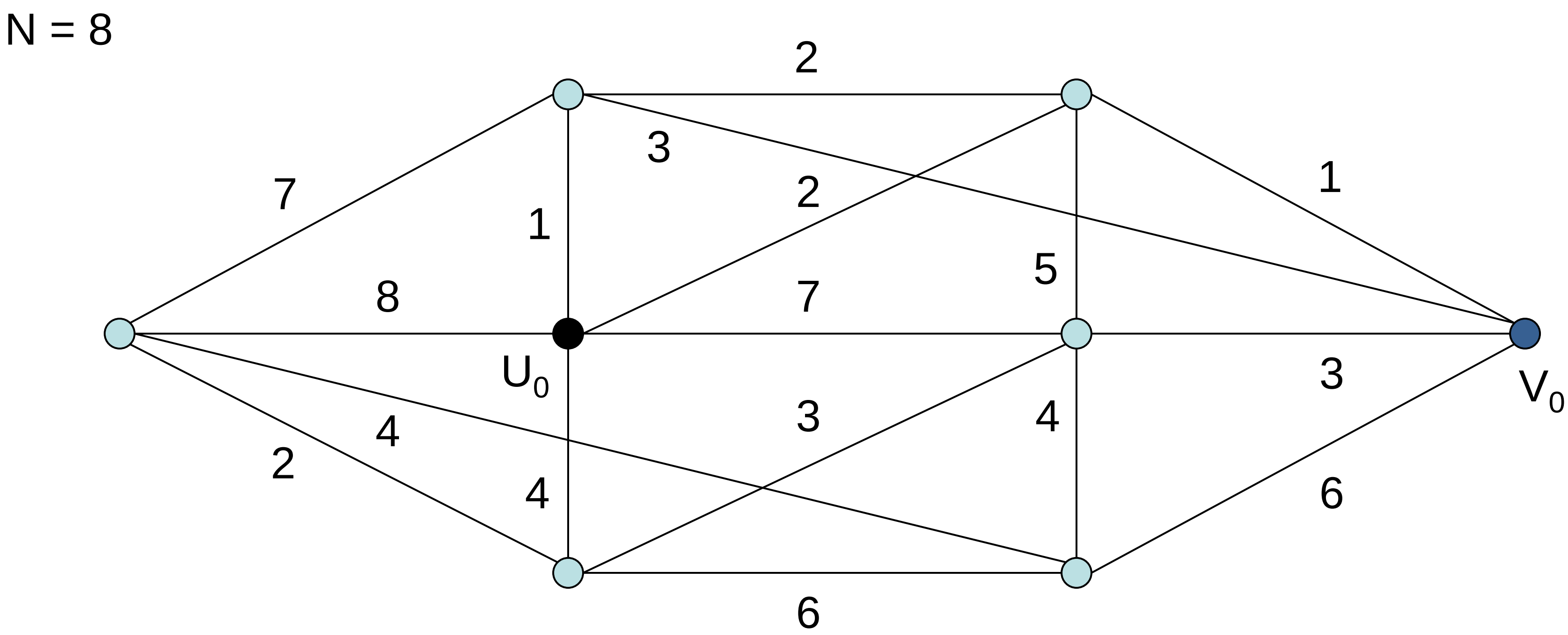
The Algorithm:

It.1) Find the nearest (i.e. linked through minimum weight) node to U_0, include it in S_1. U_1.

It.i) Find the node in S_{i-1}^c such that the distance (sum of all weights) from the source is minimal, extending by one edge from one of the nodes included in S_{i-1}. Include it in S_i. U_i.

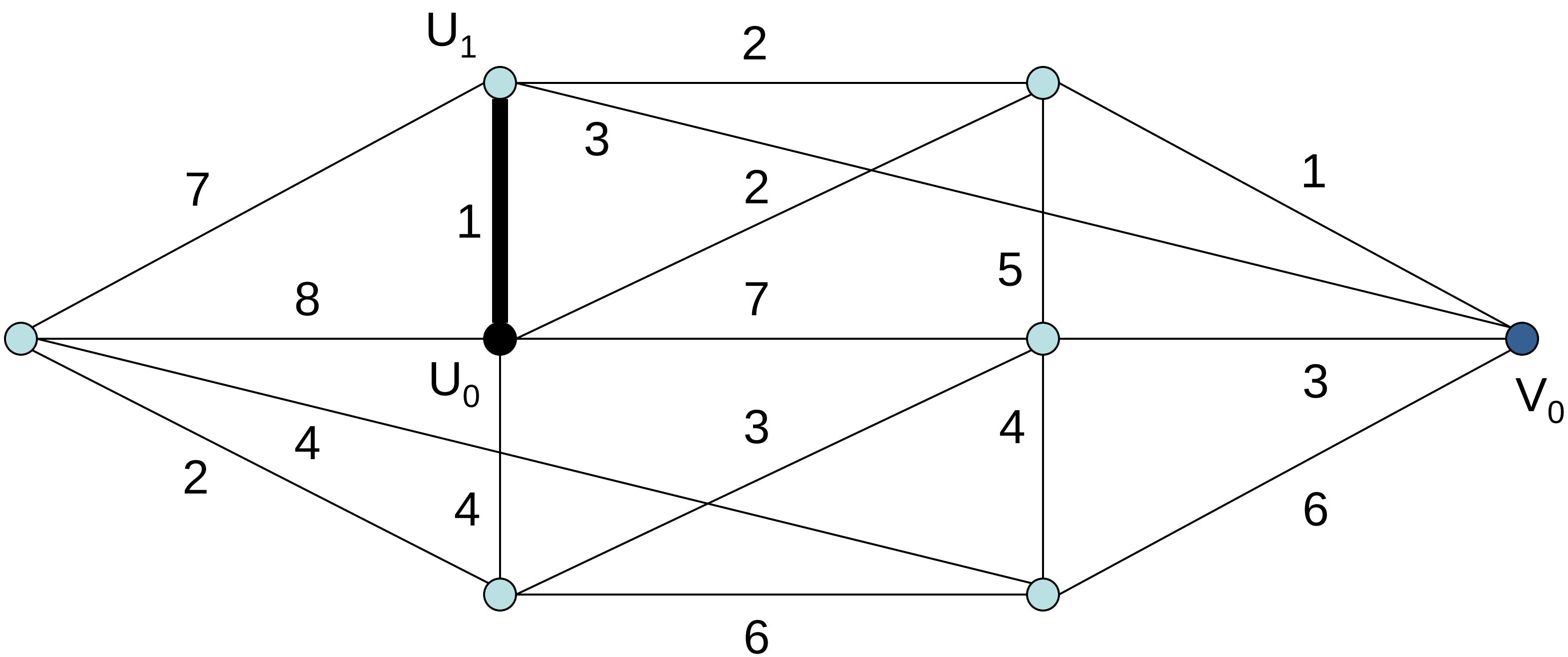
When i reaches N-1, the algorithm ends.

Dijkstra: Example



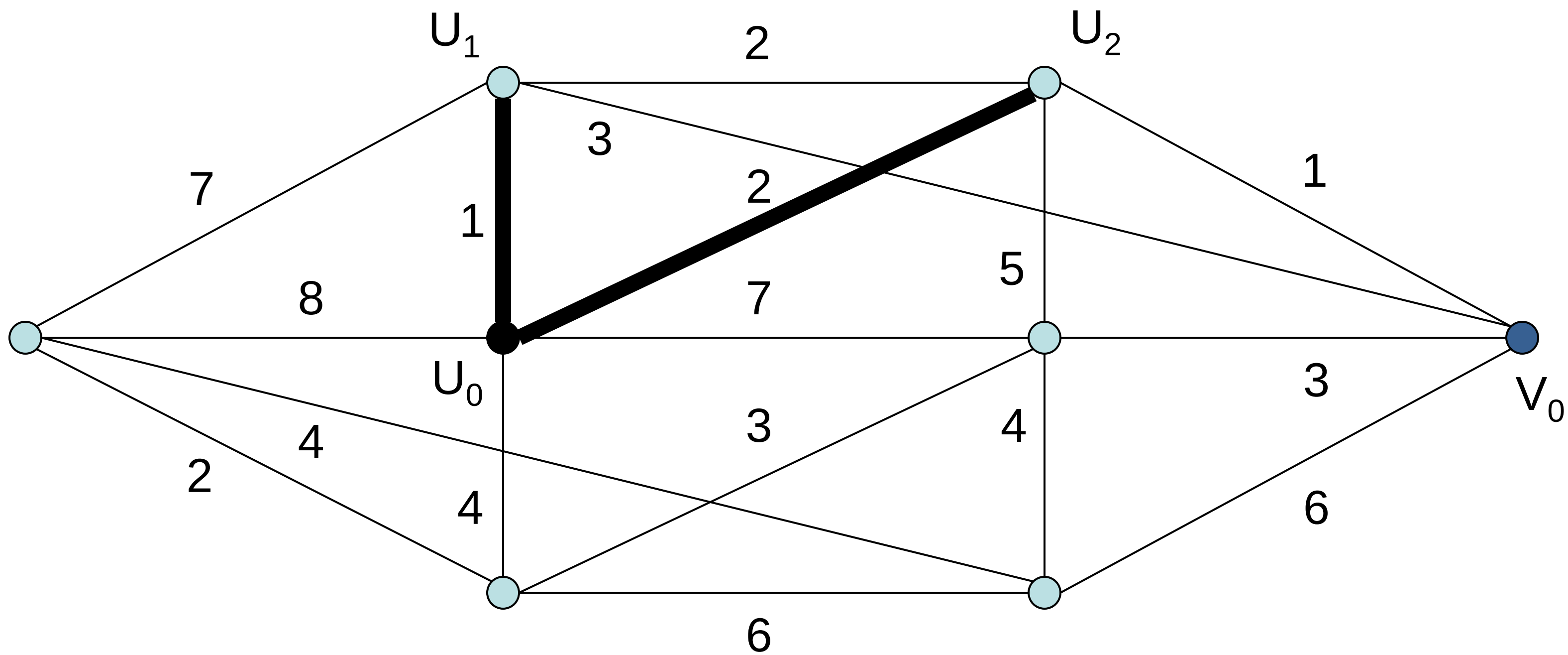
Dijkstra: Example

Iteration 1



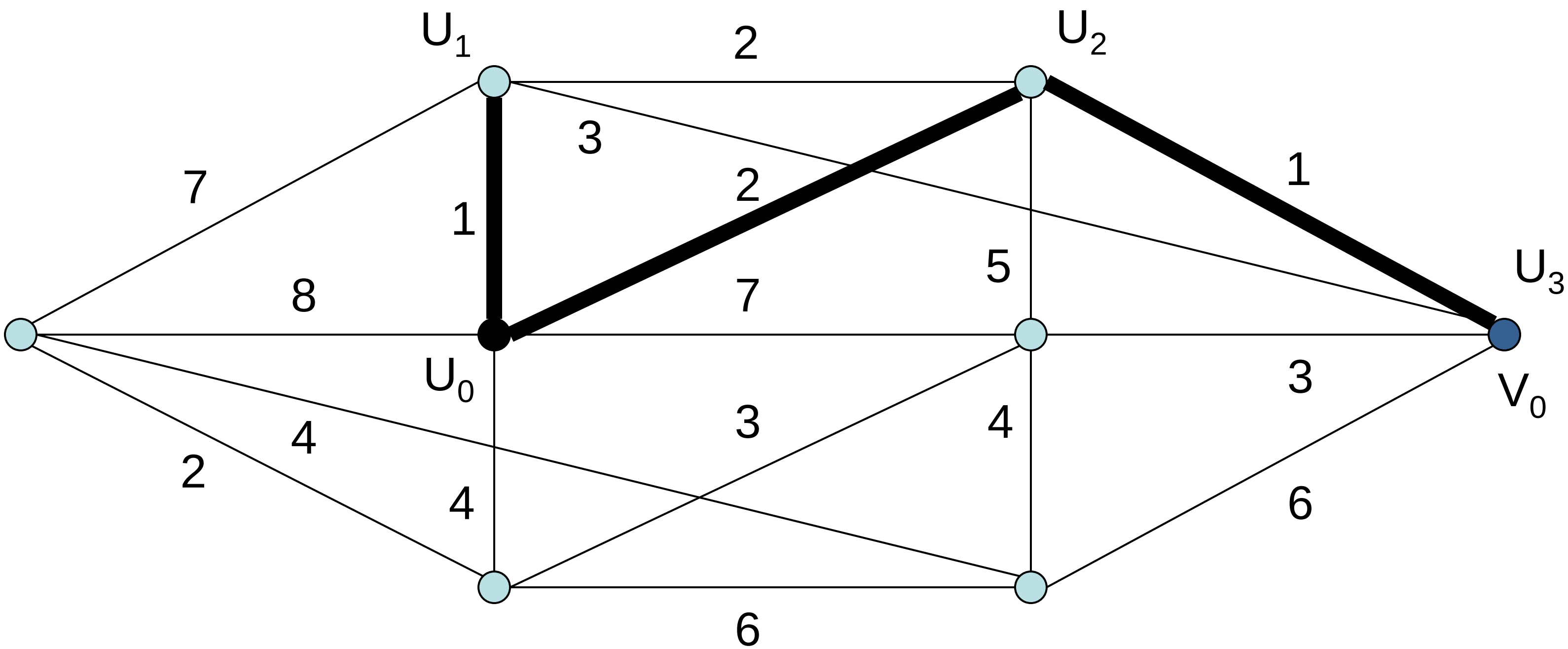
Dijkstra: Example

Iteration 2



Dijkstra: Example

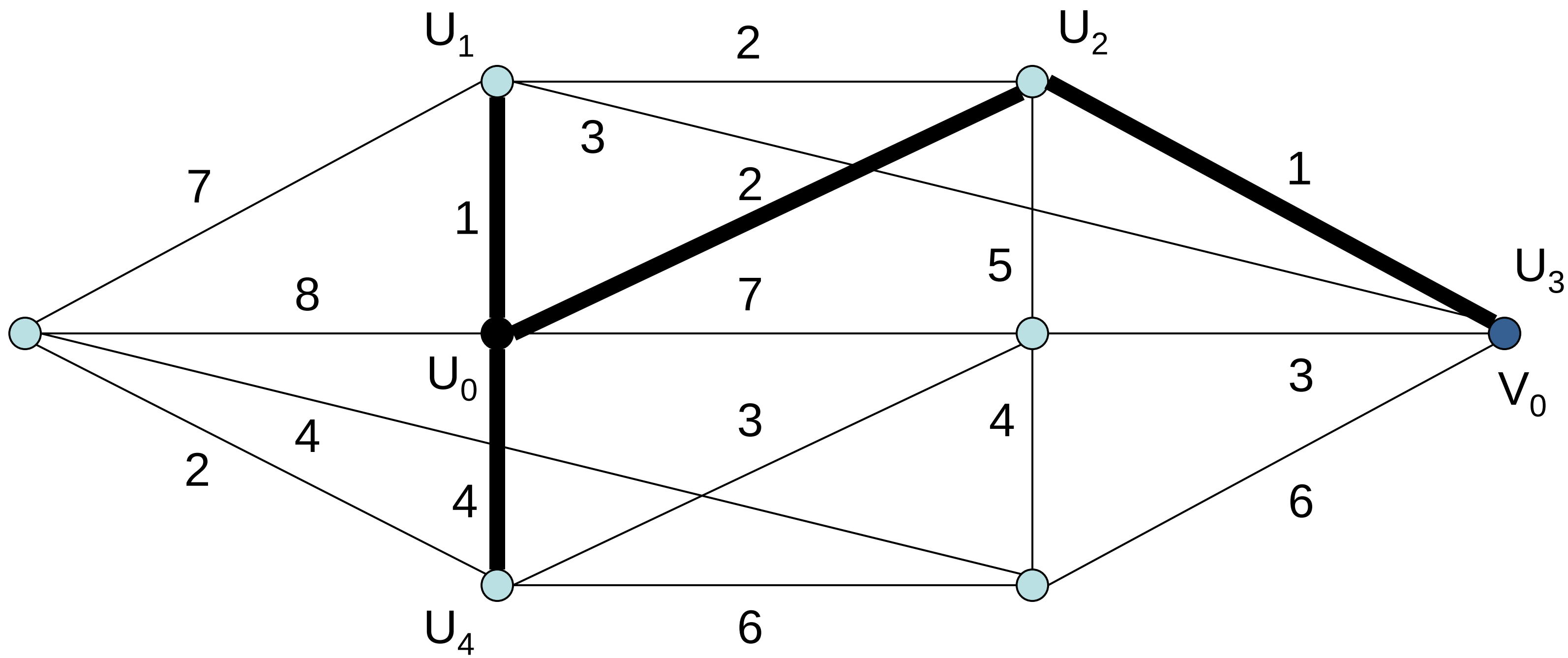
Iteration 3



Note: the algorithm could be stopped now

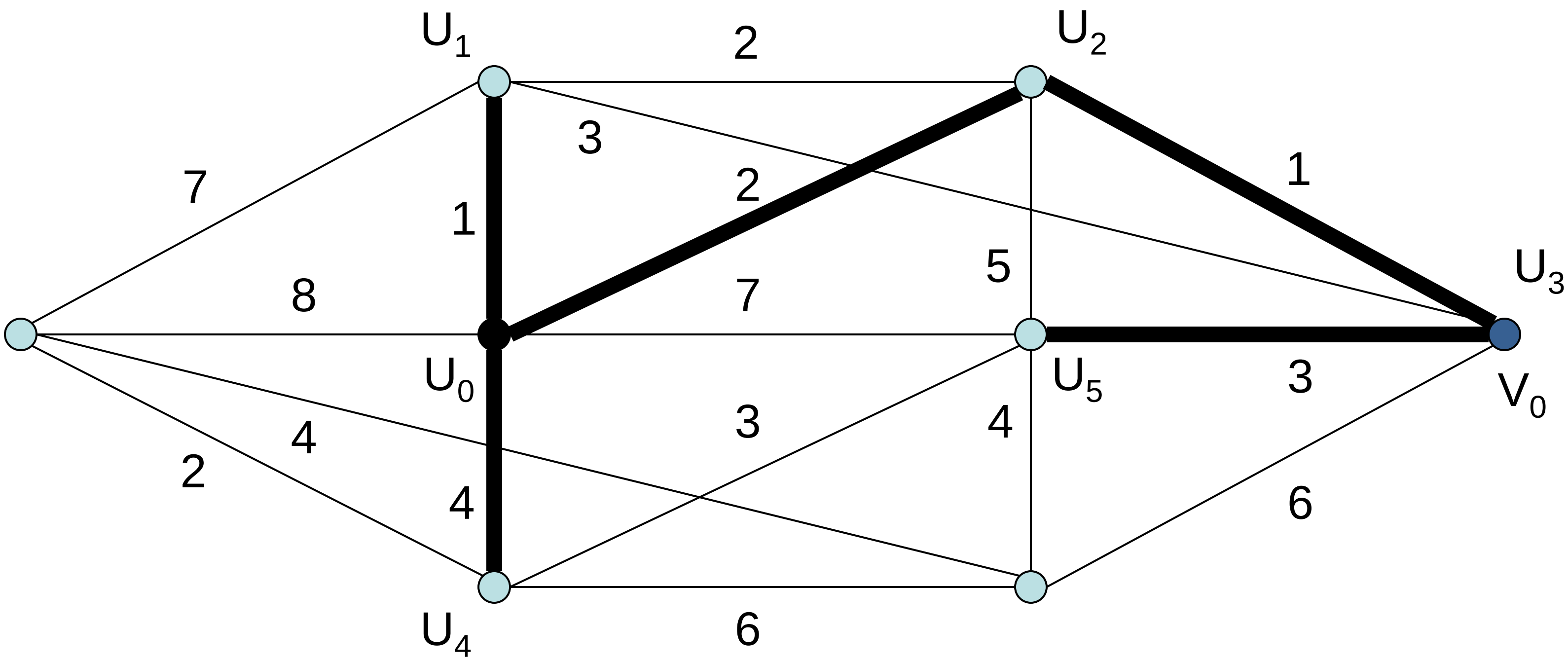
Dijkstra: Example

Iteration 4



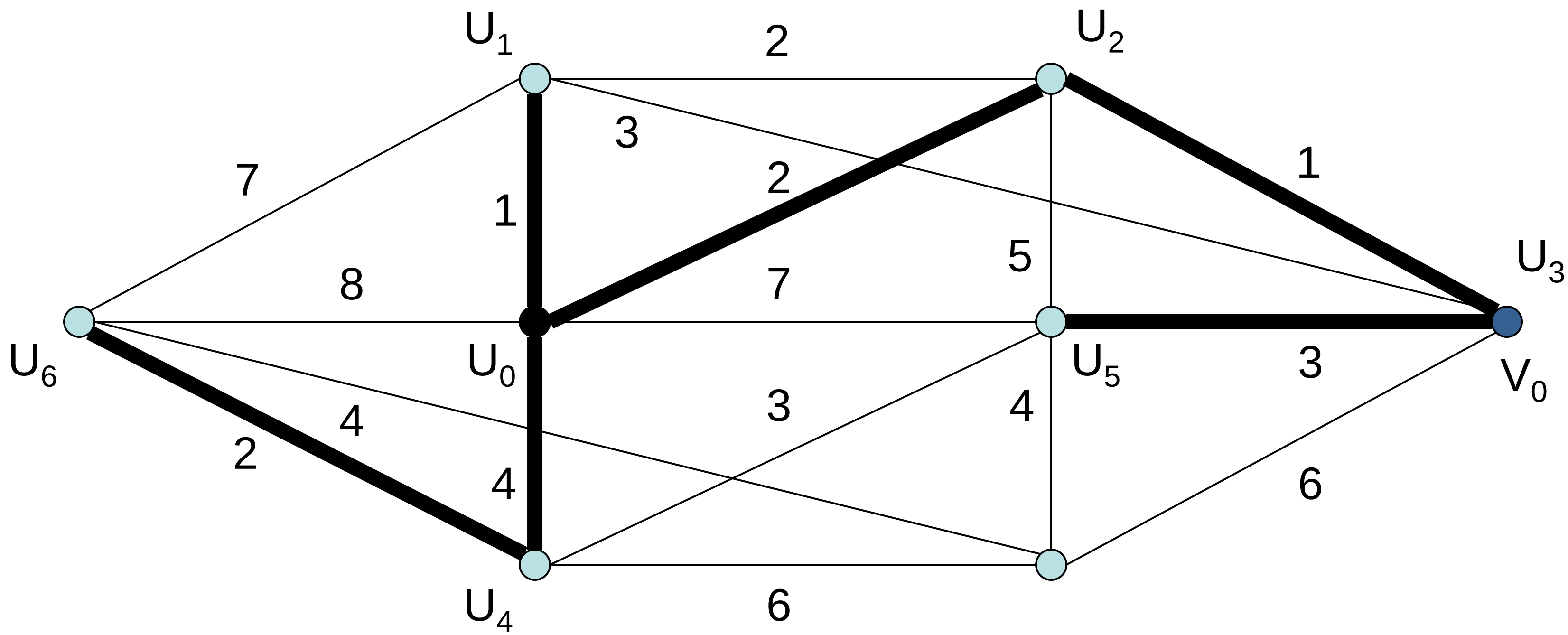
Dijkstra: Example

Iteration 5



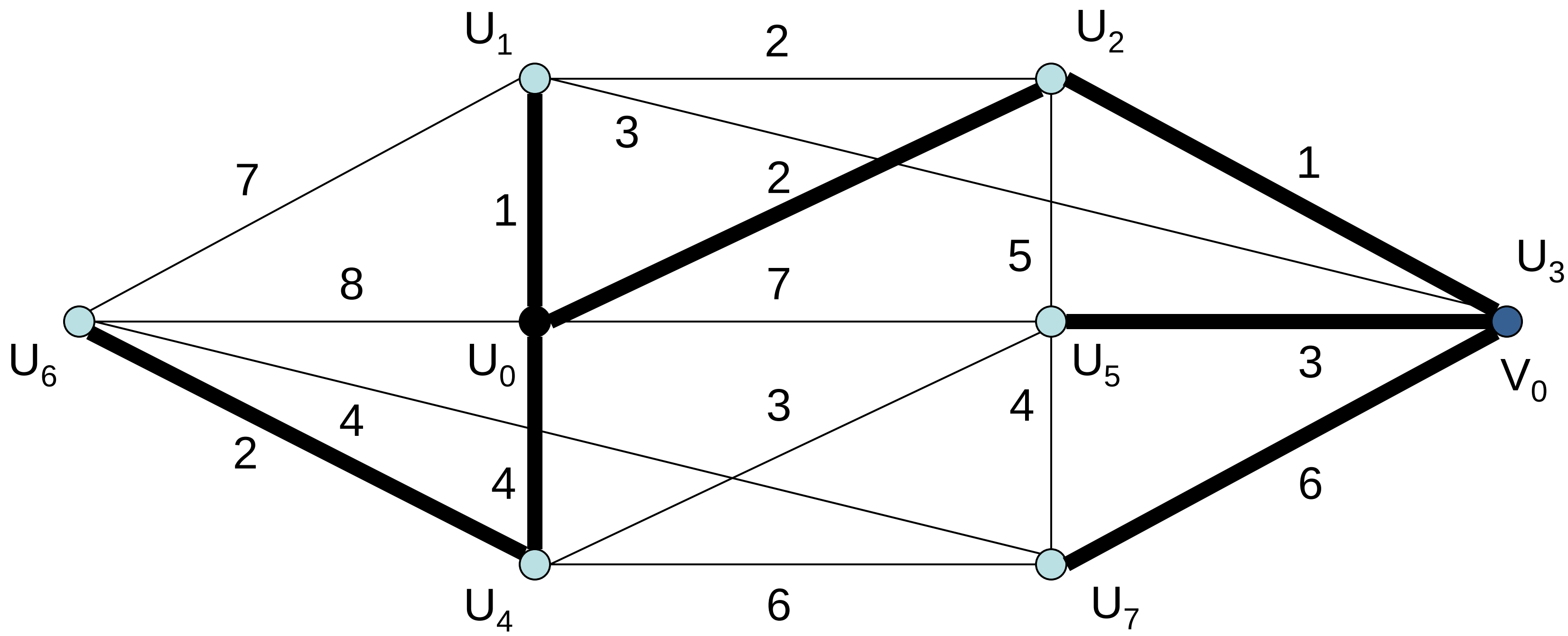
Dijkstra: Example

Iteration 6



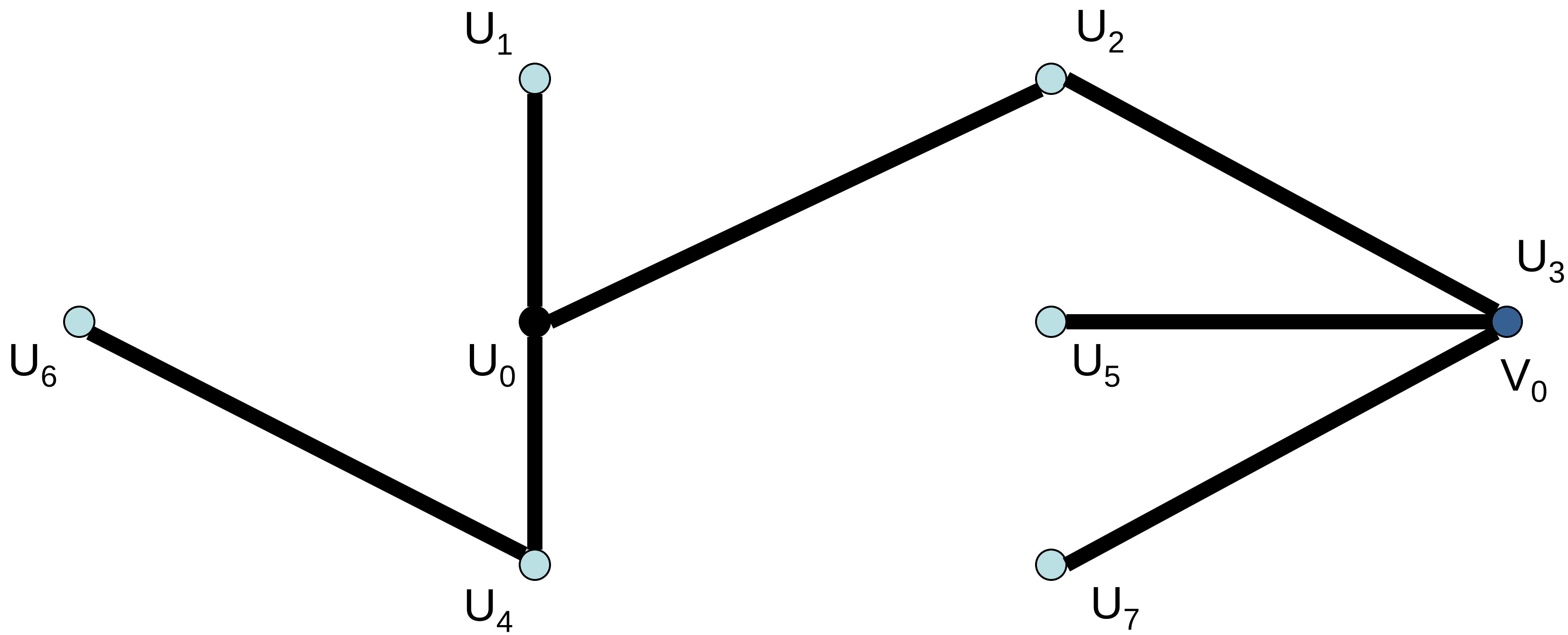
Dijkstra: Example

Iteration 7



The algorithm ends.

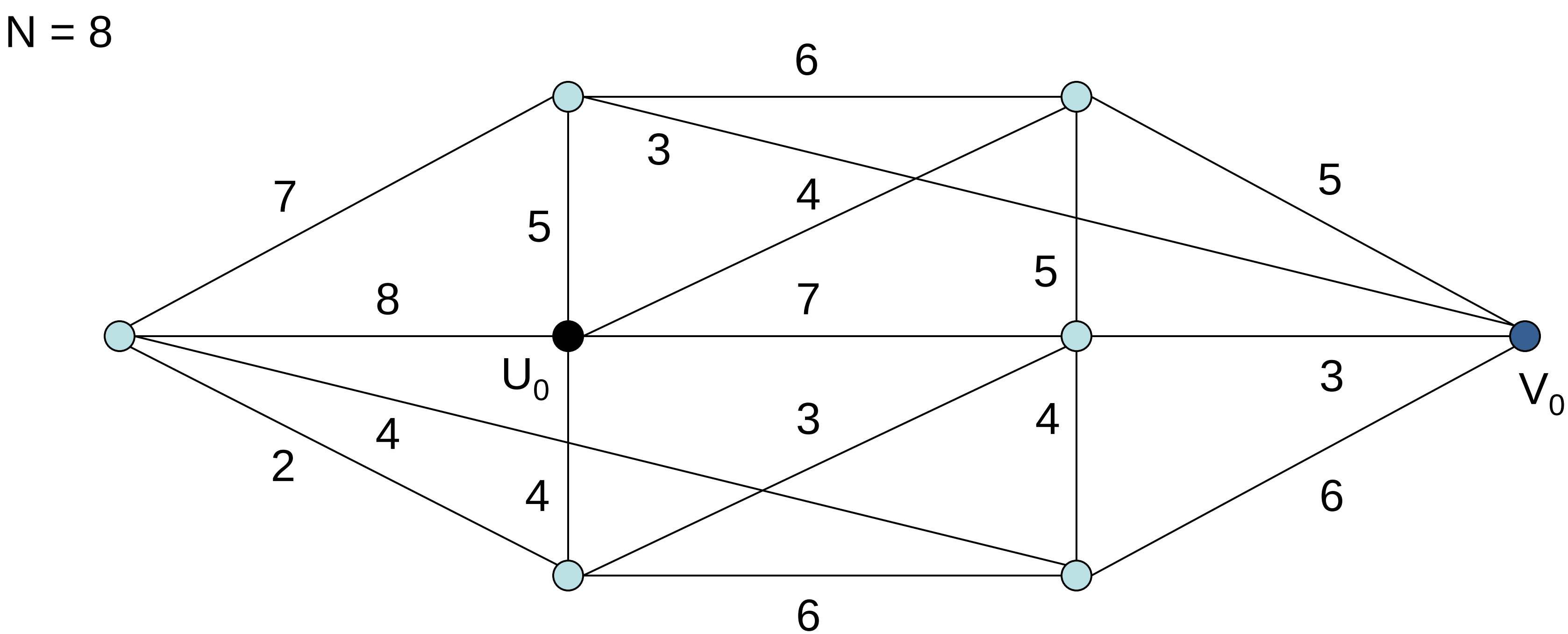
Dijkstra: Example



The final tree has height three (some leaves are level three nodes).

Exercise MNG#2

Determine the shortest path for node A to reach node B in the weighted graph below.



Exercise MNG#3

Determine the shortest path for node A to reach node B in the network below. Assume that received powers at each node can be computed according to free space assumptions, with unitary antenna gains at all nodes, same level of transmit power -10 dBm, receiver sensitivity set as -89 dBm and wavelength of 12 cm. Weights are equal to the received power in dBm, with change of the sign (if above sensitivity, otherwise zero). Repeat with weights set as square of distances. Repeat the exercise with transmit power set at 10 dBm.