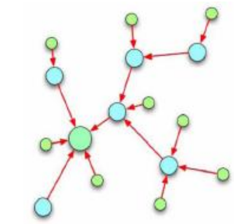
WSN Lab at the
University of Bologna



# Wireless Sensor and Actuator Networks:
## *Technologies, Analysis and Design*

# Network Connectivity for WSANs

**Roberto Verdone**

**roberto.verdone@unibo.it**
**http://www.robertoverdone.org**

# Topology Control versus Connectivity Control

**Topology Control** aims at controlling the set of links that connect couples of nodes, in order to simplify routing of messages / allow routing of messages between all pairs of nodes.

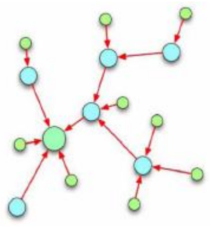
The ***physical*** **topology** of a network can be controlled through physical layer, in most cases power control techniques are used.

The ***logical*** **topology** of a network is controlled by entities working at layer 2 and 3, and is based on a **reduced set of links** wrt the physical topology.

Logical topologies can be *flat* or ***hierarchical*** according to the roles assigned to nodes.

**Connectivity Theory** aims at describing the potential topologies of a networks, assuming nodes are randomly distributed over space.

It deals mainly with physical topologies, but can be extended
to some aspects of logical topologies taking non-electromagnetic aspects into account like for instance capacities, interference, etc.



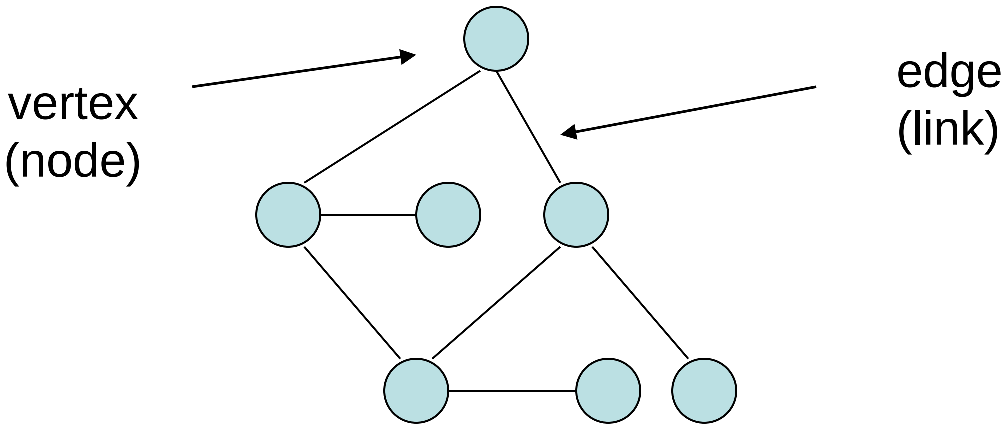
# Outline

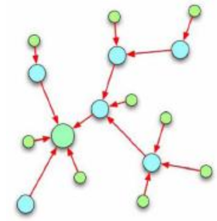
1. **Network Topologies in WSNs**
2. **Connectivity Theory: Preliminaries**
3. **Critical Transmission Range**
4. **Connectivity Over an Unlimited Region**
5. **Connectivity for WSNs**
6. **Connectivity Over Limited Regions for WSNs**

# Background: Elements of Graph Theory

**Graph**
A set of items connected by *edges*. Each item is called a *vertex* or node.
Formally, a graph is a *set* of vertices and a *binary relation* between them, adjacency.
**Formal Definition:** A graph G can be defined as a pair (V,E),
where V is a set of vertices, and E is a set of edges between the vertices
E = {(u,v) | u, v in V}.

If the graph is **undirected**, the adjacency relation defined by the edges is *symmetric*,
or E = {{u,v} | u, v in V} (sets of vertices rather than ordered pairs).



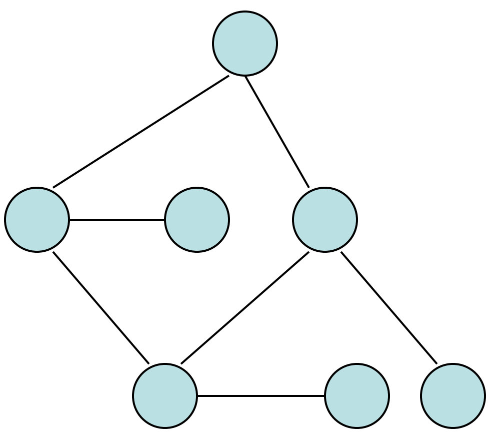


# Background: Elements of Graph Theory

**Geometric Graph (GG)**
Vertices have a geometric location in $\mathbf{R}^d$. In the following we assume d=2.

**Random Graph**
Edges between pairs of nodes exist according to random statistics.

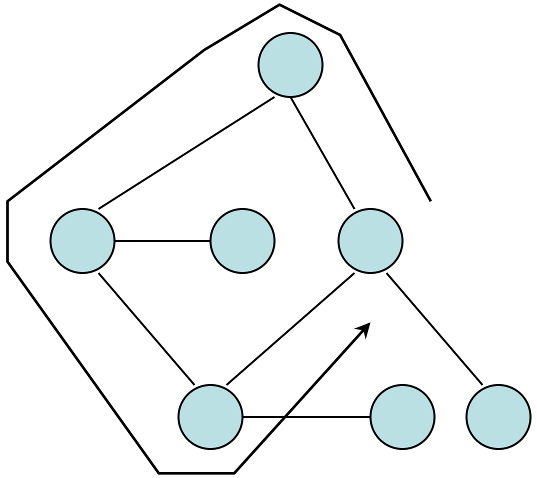
**Geometric Random Graph (GRG)**
Random Graph where edges exist according to proximity relation between nodes and nodes are in unknown positions.
In GRGs, the set of nodes is normally finite, and their number deterministically known.



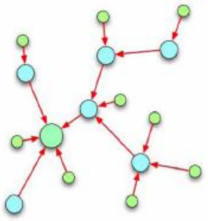
# Background: Elements of Graph Theory

**Complete Graph**
An *undirected graph* with an *edge* between every pair of *vertices*

**Acyclic Graph**
A *graph* with no *path* that starts and ends at the same *vertex*.



Not complete
Not acyclic

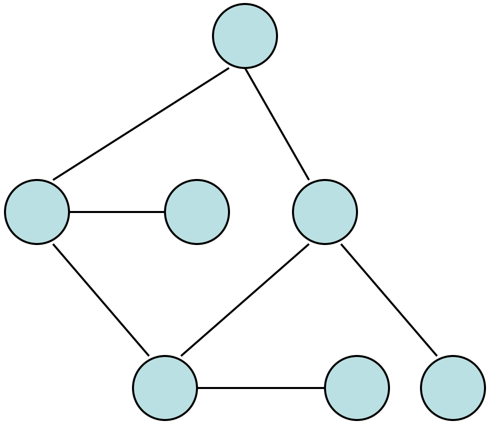


# Background: Elements of Graph Theory

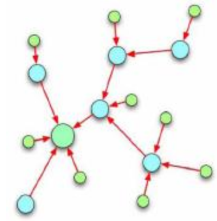
**Connected Graph**
An *undirected graph* that has a *path* between every pair of *vertices*.

**Edge Connectivity**
The smallest number of *edges* whose deletion will cause a *connected graph* to not be connected.

**Node Connectivity**
The smallest number of *vertices* whose deletion causes a *connected graph* to not be connected.



Connected
EC=1
NC=1



# Background: Elements of Graph Theory
# The Communication Graph

**A Network is a pair (N,L)**
N is a set of wireless nodes, of size n. Assume they are located in an unit square.
L is the function mapping every node u to a position L(u).

**A Range Assignment** is a function assigning to every node u a transmit range RA(u).

**The Communication Graph** is the directed graph (G,E) where the directed edge (u,v) exists if the Euclidean distance between u and v is less or equal than RA(u).
In this case v is neighbour to u. If u is also neighbour to v for all pairs (u,v), the Communication Graph is undirected and all links are symmetrical.

A RA for a Network is **connecting** if the correspondent CG is connected.

A RA where all nodes have the same transmit range is said **homogeneous**.
If the value of the transmit range R is relevant, than the RA is said **R-homogeneous**.

# Background: Elements of Graph Theory

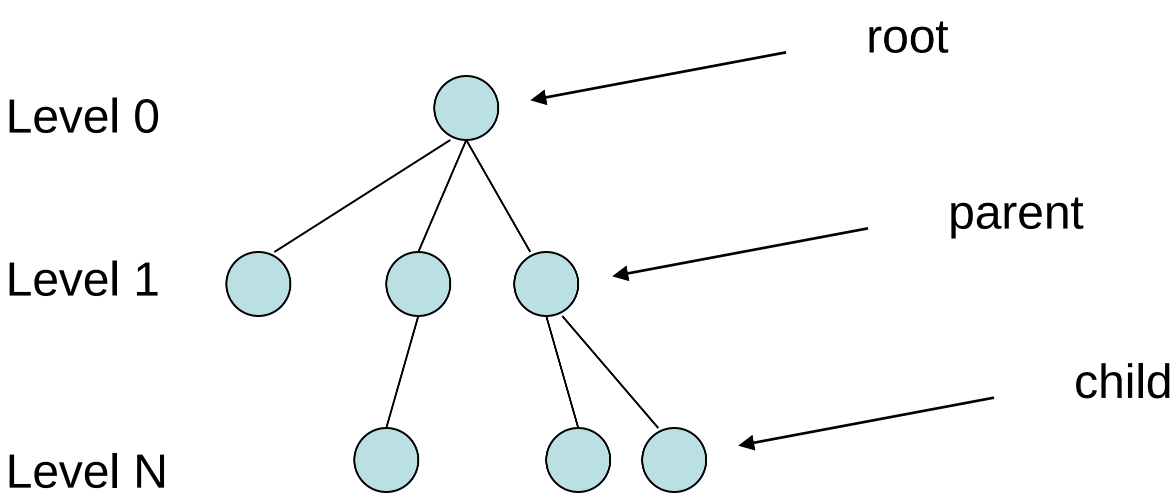
**Tree**

A *connected*, *undirected*, *acyclic graph*.

It is a data structure accessed beginning at the *root* node, where each *node* is either a *leaf* or an *internal node*.

An internal node has one or more *child* nodes and is called the *parent* of its child nodes. All children of the same node are *siblings*.

Contrary to a physical tree, the root is usually depicted at the top of the structure, and the leaves are depicted at the bottom.



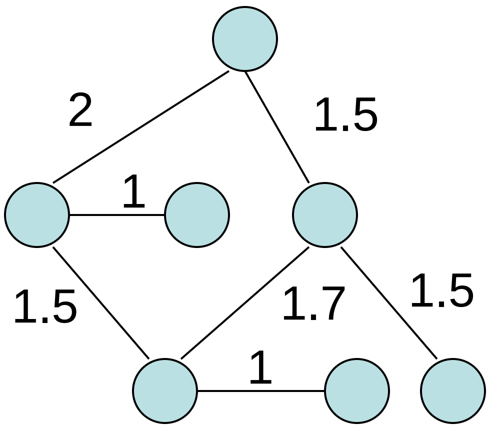
# Background: Elements of Graph Theory

**Weighted Graph**

A *graph* having a weight, or number, associated with each *edge*

**Euclidean Tree**

A *tree* in a weighted GG where weights are assigned to edges based on Euclidean distances.

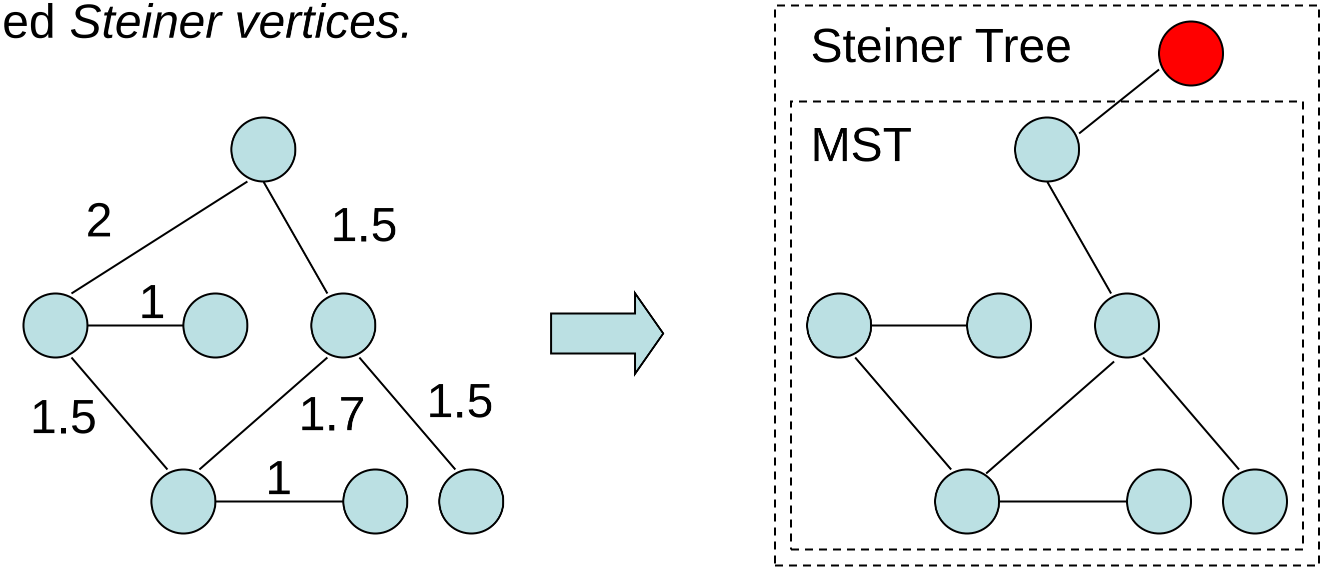


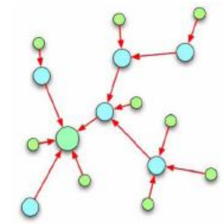
# Background: Elements of Graph Theory

**Spanning Tree**
A connected, acyclic subgraph containing all the vertices of a graph

**Minimum Spanning Tree (MST)**
A minimum-weight tree in a weighted graph which contains all of the graph's vertices.

**Steiner Tree**
A minimum-weight *tree* connecting a designated set of *vertices*, called *terminals*, in an *undirected*, *weighted graph*. The tree may include non-terminals, which are called *Steiner vertices.*



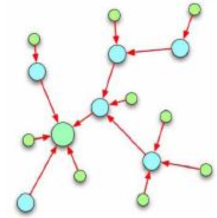


# Section 1
# Network Topologies in WSANs

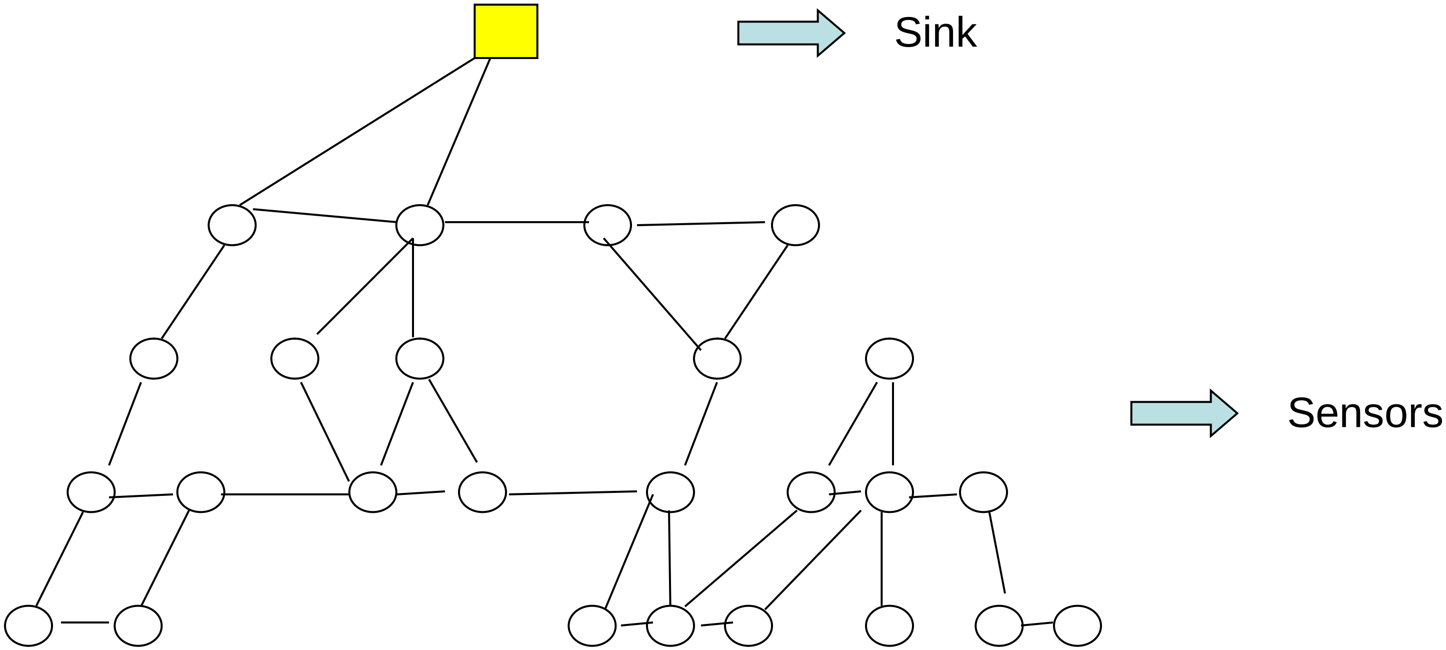
**Flat**

**Trees**

**Optimised Tree Design**

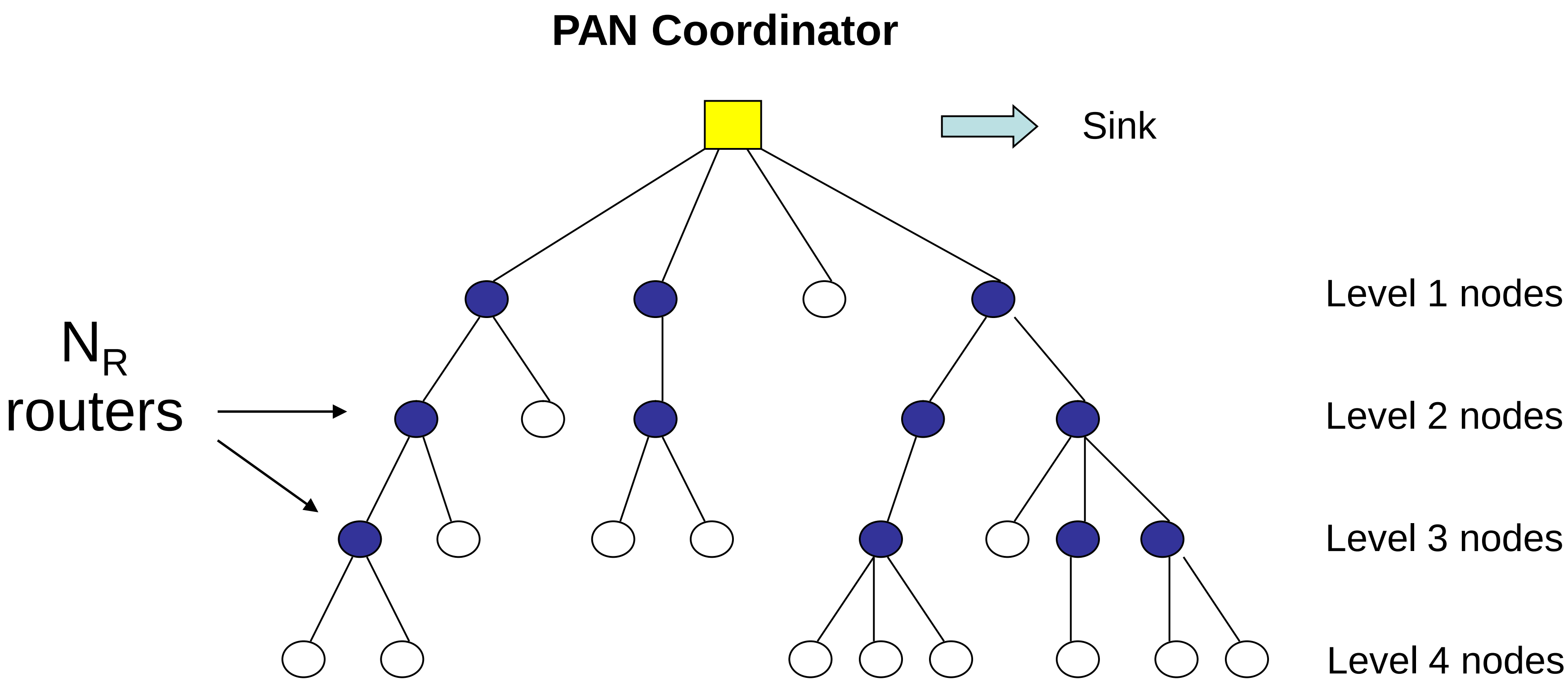


# Flat




With 802.15.4, the NBE mode should be used.

# Trees




With 802.15.4 BE mode: $2^{BO} \geq (N_R + 1) \cdot 2^{SO}$.

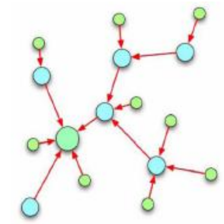
# Trees

## Pros

Routing is simple
Local Addressing is a simple task
Few parameters allow control of the topology
Data aggregation strategies are simplified

## Cons

Large average delays
Critical Links



**Owing to the type of information flow, trees (hierarchical topologies) are the most appropriate.**

**For the sake of energy efficiency, the role of nodes must be often rotated.**

**Topologies are thus inherently dynamic.**

**Some applications include moving sensors and the topologies are consequently very dynamic.**

**Efficiency of Topology Reconfiguration mechanisms is a key issue.**

# Section 2
# Connectivity Theory: Preliminaries

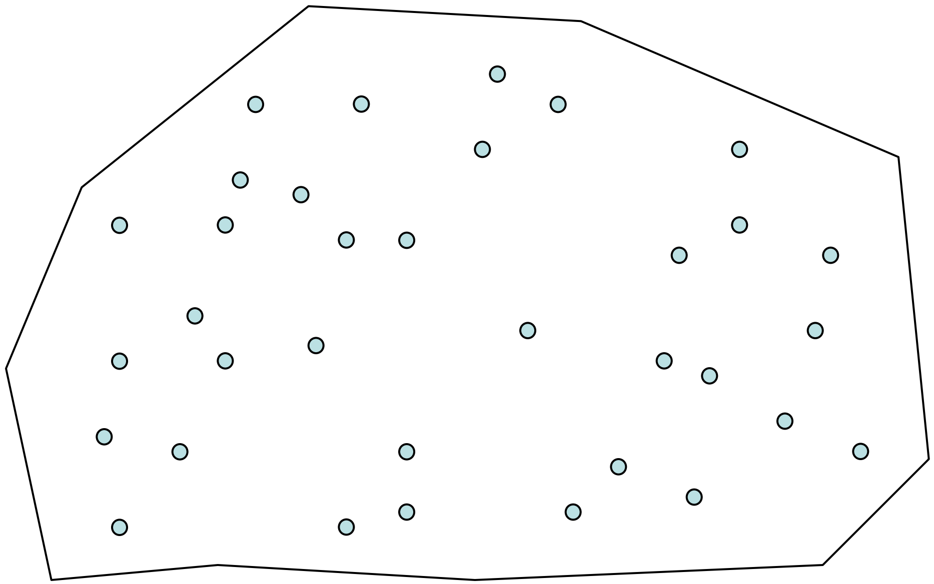
**What does it aim to**
**Link Connectivity**
**Full Connectivity**
**Critical Transmission Range**

# What does it aim to

In networks formed by large numbers of nodes distributed according to some **statistics** over a limited or unlimited region of $\mathbf{R}^d$, Connectivity Theory aims at describing the potential set of links that can connect nodes to each other, subject to some constraints from the physical viewpoint (power budget, or radio resource limitations).

It studies network properties

d = 2 is considered here.

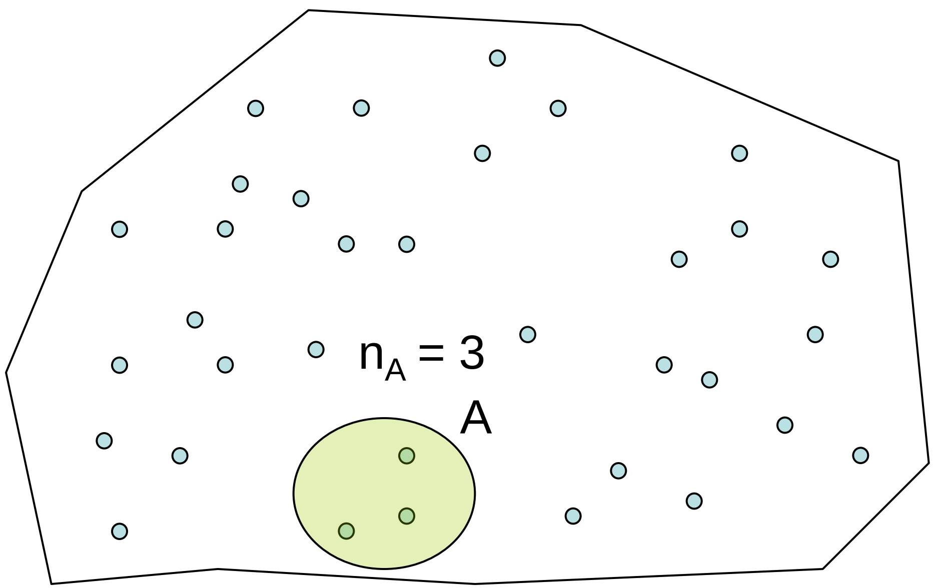


# Poisson Point Process (PPP)

Given a (sub-)region A, the number of nodes in A, $n_A$, is Poisson, with mean $N_A = \rho A$ where $\rho$ is node density.
This is equivalent to **random and uniform independent positions of nodes**.

$\text{Prob}(n_A = n) = e^{-\rho A} (\rho A)^n / n! = e^{-N_A} (N_A)^n / n!$
The probability that the region is empty is $\text{Prob}(n_A = 0) = e^{-\rho A} = e^{-N_A}$

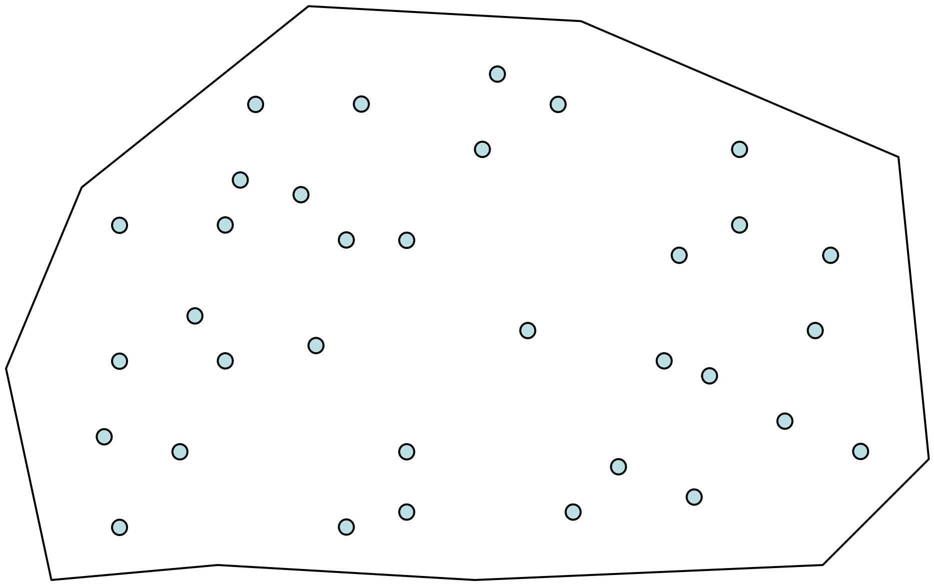
PPPs may be considered either on bounded or unbounded regions.



# What does it aim to
# Examples

What is the probability that the network is **fully connected**
(i.e. every node can reach any other node through any number of hops)
when nodes are distributed in the region according to a PPP with intensity $\rho$.

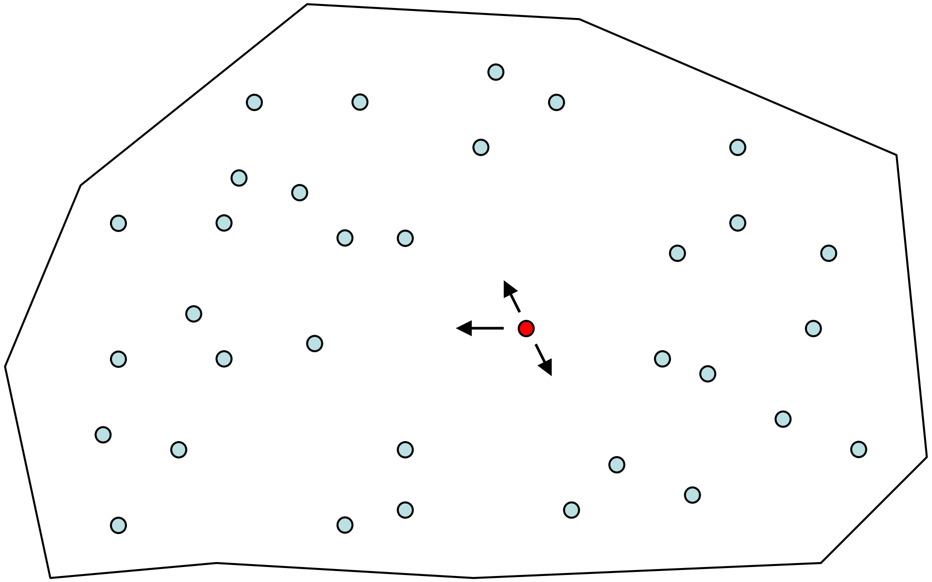
Relevant in ad hoc nets.



# What does it aim to
# Examples

What is the probability that **a node is isolated**
when nodes are distributed in the region according to a PPP with intensity $\rho$.

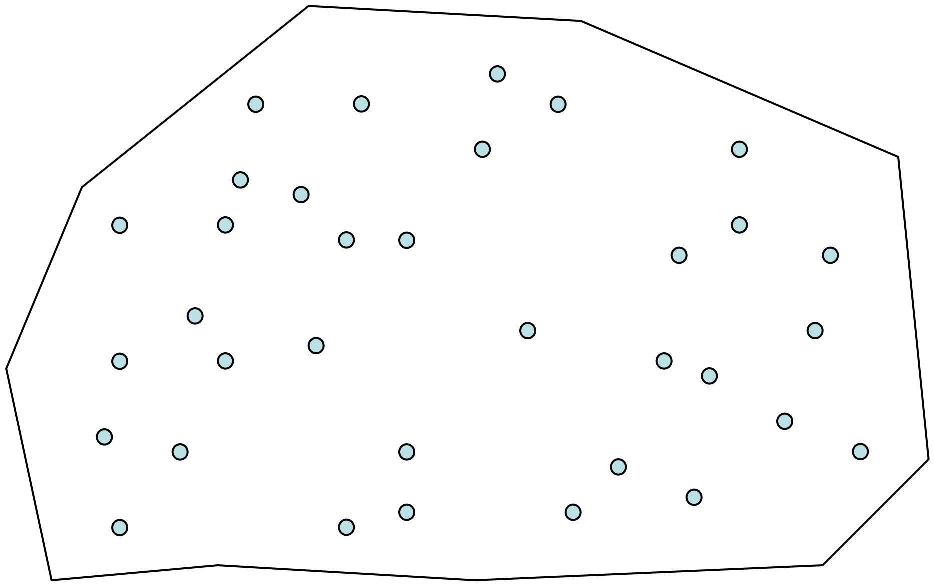
Relevant in WSNs.



# Regions

In the literature, typically three types of statistical scenarios are considered:

- Square of unit side (or disk of unit radius) with nodes distributed in the region according to a PPP with intensity $\rho$;
- A PPP with intensity $\rho$ over an unbounded region;
- Square of unit side (or disk of unit radius) with N (N is given) nodes uniformly distributed at random in the region.

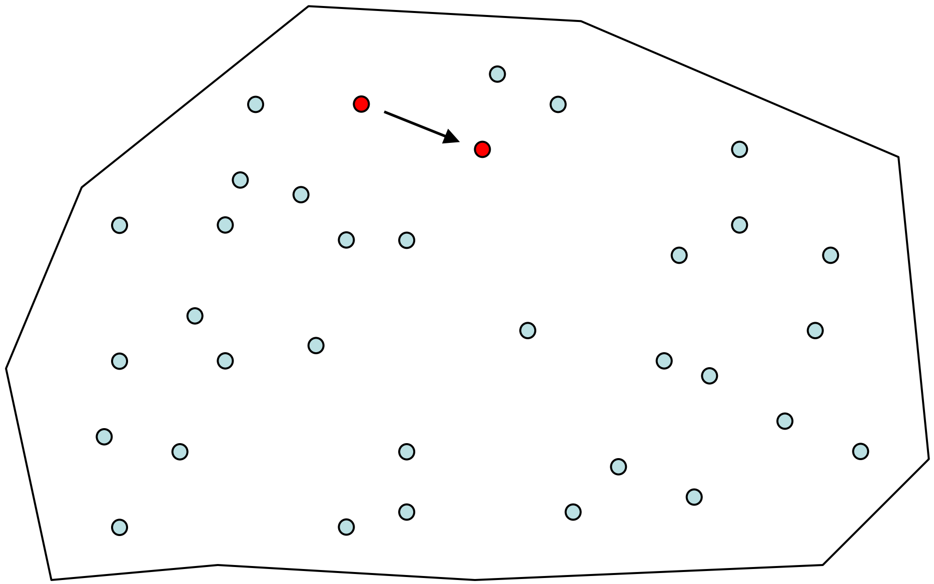


# Link Connectivity

To study (network) connectivity, one has to define link connectivity properties.

Different models in the literature, all considering narrowband systems.

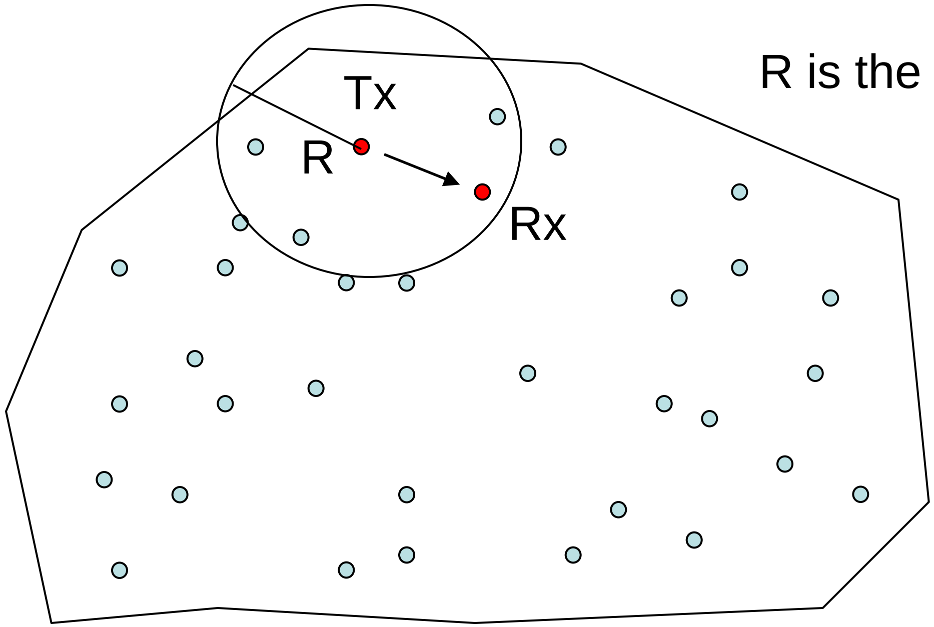
Can be used for 802.15.4, Bluetooth.



# Link Connectivity

Model 1 (deterministic distance - dependent model). The most widely used.

Rx is connected to Tx if SNR is above minimum threshold $\alpha$, i.e.
if received power $P_r$ is above minimum threshold $P_{rmin}$, i.e.
if power loss L is smaller than maximum value $L_{th}$
and $L = k_0 + k_d \log r$ i.e.
if distance r is below a given maximum value R




R is the transmitting range.

# Link Connectivity

Model 2 (random distance - dependent model). Used in the following.

Rx is connected to Tx

if SNR is above minimum threshold $\alpha$, i.e.

if received power $P_r$ is above minimum threshold $P_{rmin}$, i.e.

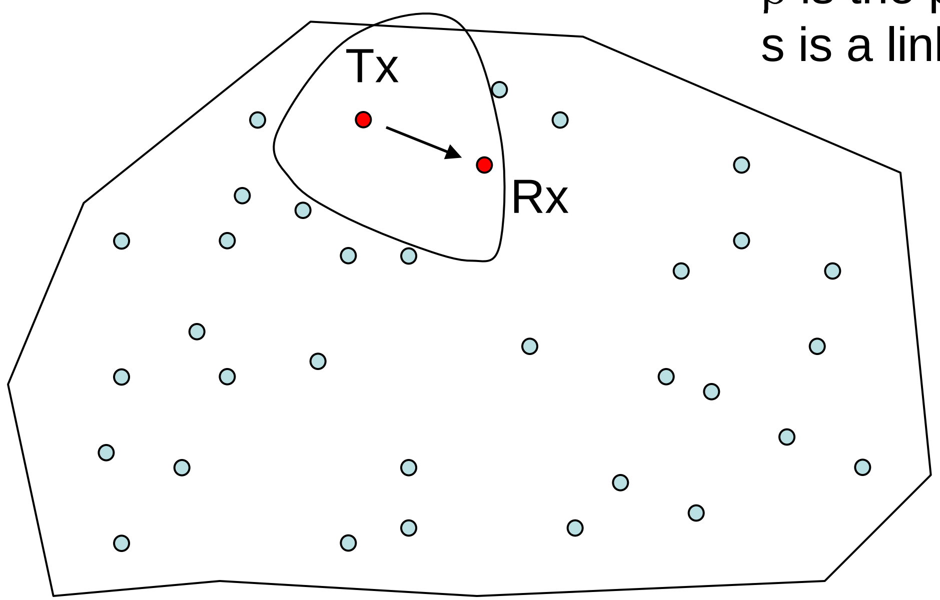
if power loss L is smaller than maximum value $L_{th}$

and $L = k_0 + k_d \log r + \mathbf{s}$

$k_d = 10\ \beta$

$\beta$ is the propagation exponent

s is a link-dependent r.v.

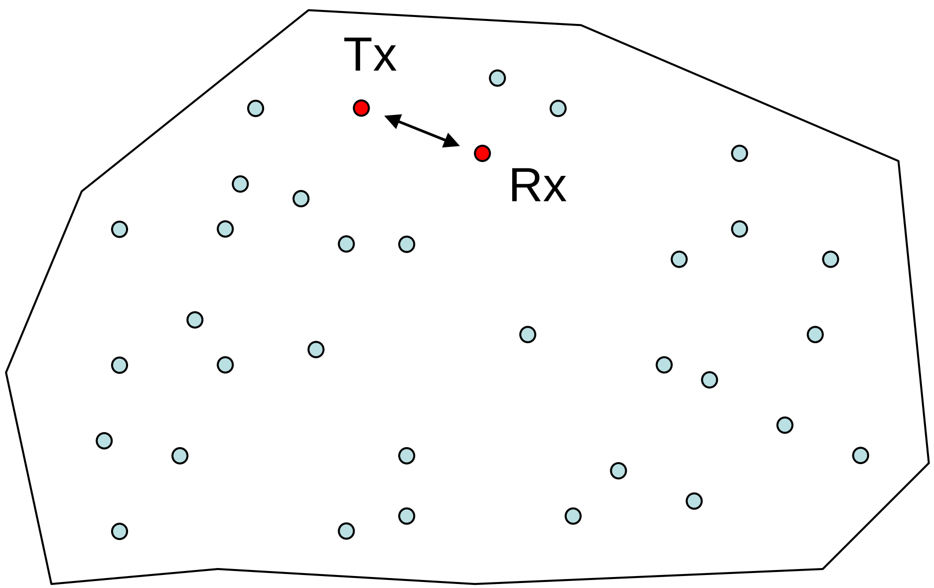


# Link Connectivity

We adopt **Model 2**, with s modelled as zero mean Gaussian r.v. with variance $\sigma^2$.

$L = k_0 + k_1 \ln r + s$ $\qquad$ $k_1 = k_d / \ln 10$

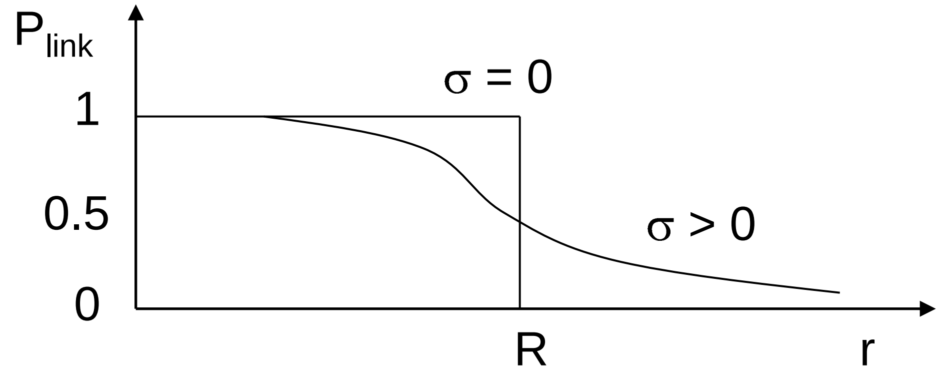
When variance of s is zero, Model 2 converges to Model 1.

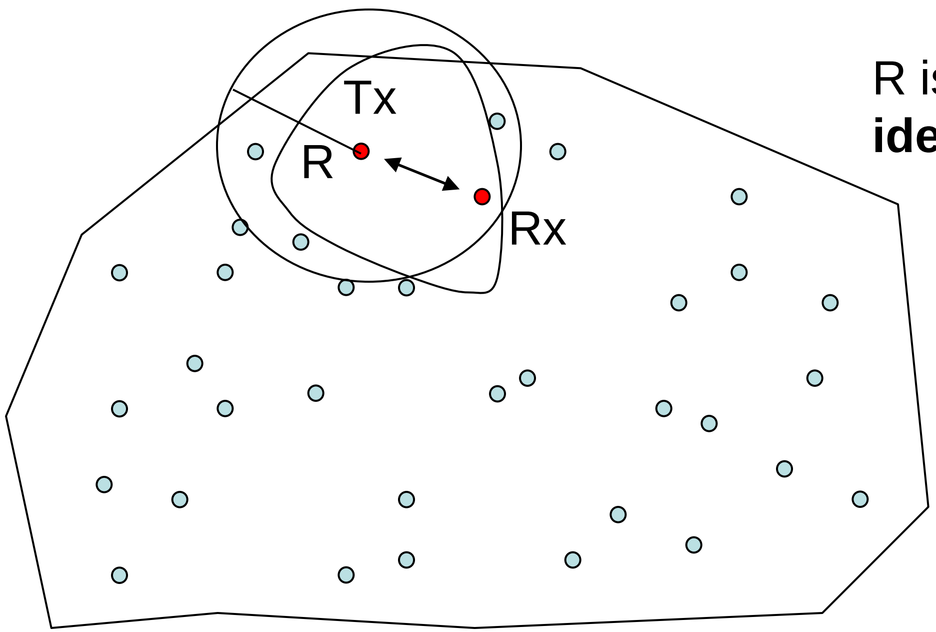


# Link Connectivity

$$P_{link} = \Pr\{L(dB) < L_{th}\} = \Pr\{s < L_{th} - k_0 - k_1 \ln(r)\} =$$

$$= \frac{1}{2} + \frac{1}{2} erf\left(\frac{L_{th} - k_0 - k_1 \ln(r)}{\sqrt{2}\sigma}\right)$$




R is the
**ideal** transmission range

$R = \exp((L_{th} - k_0)/k_1)$

# Full Connectivity

Traditional definition:

*A network is fully connected if there exists any path (sequence of hops) between every pair of nodes*.

This definition is compliant with the objective of ad hoc networks, i.e. to allow every node being in contact with any other node.

But this is not the goal of a WSN.

In WSNs, nodes (sensors) want to transmit their samples to a given node, namely, the sink (or any node in a given set, in the case of multi-sink networks).

Definition more suitable for WSNs:

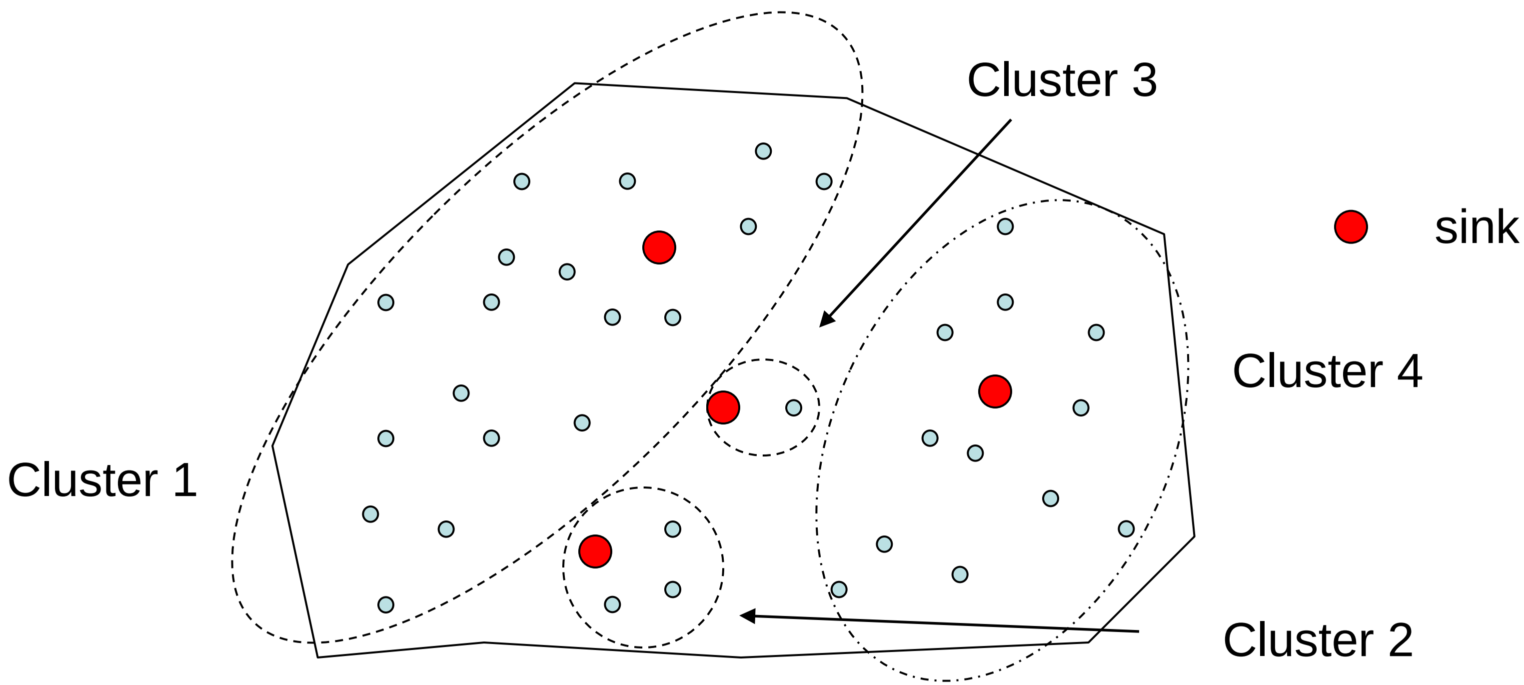
*A WSN is fully connected if all nodes can report their samples to a sink through any path.*

# Full Connectivity

Note that under such new definition, a WSN can be fully connected even if some nodes can not reach other nodes, in a multi-sink scenario.

In other words, a WSN can be fully connected even if clustered.

The difference is relevant for finite (and small) densities.



# Section 3
# Critical Transmission Range

**Critical Transmission Range**
**The Giant Component**

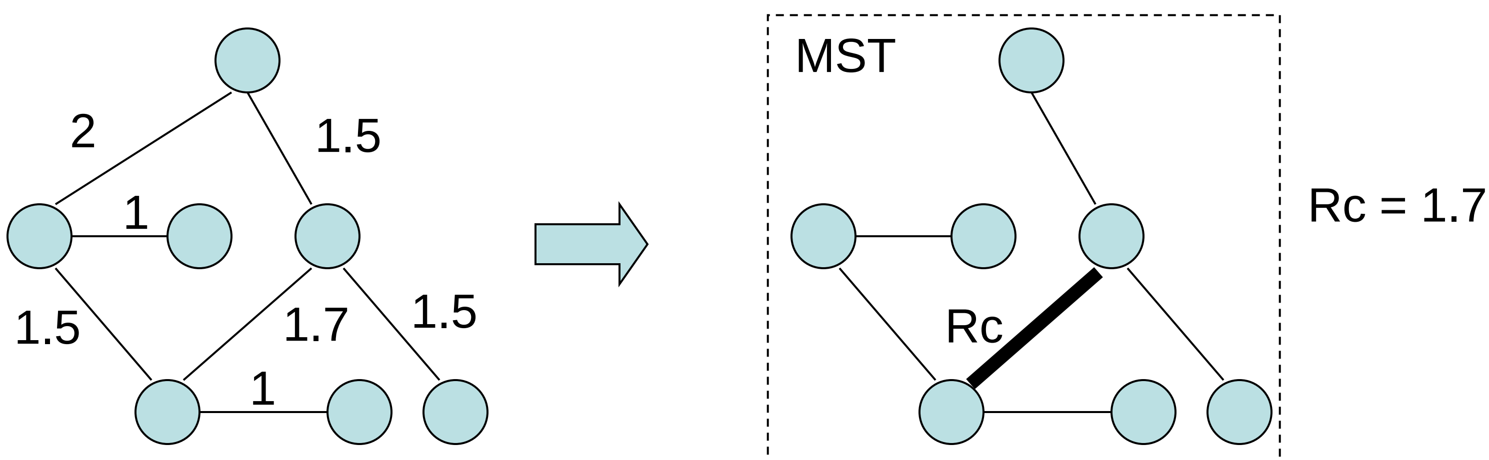
# Critical Transmission Range

**Critical Transmission Range:**
The minimum value of R for a Network s. t. the network is fully connected.

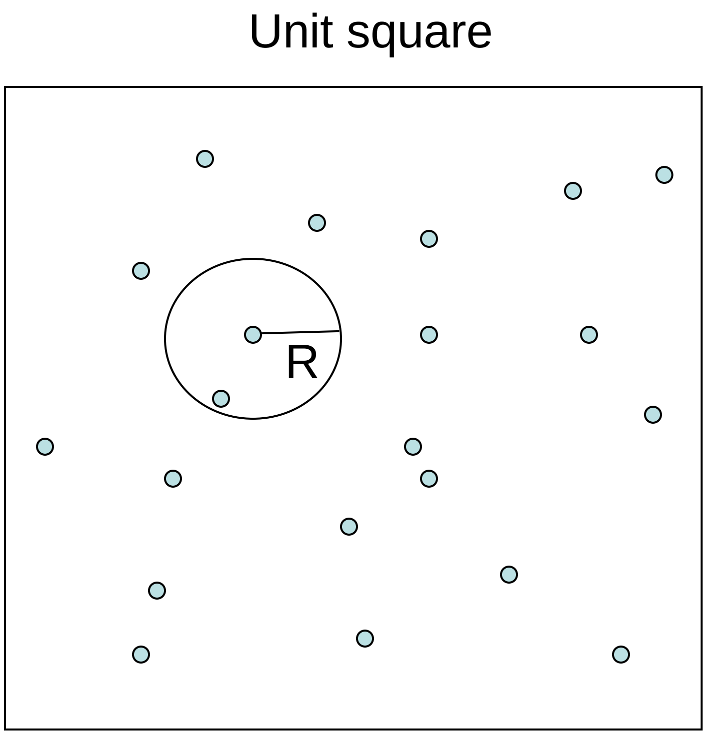
If the nodes in the Network are randomly distributed, the CTR is a random variable.

**Theorem:**

**The CTR for connectivity Rc of a Network equals the length of the longest edge of the Euclidean MST of the corresponding Communication Graph**



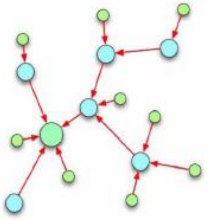
# Critical Transmission Range




n number of nodes

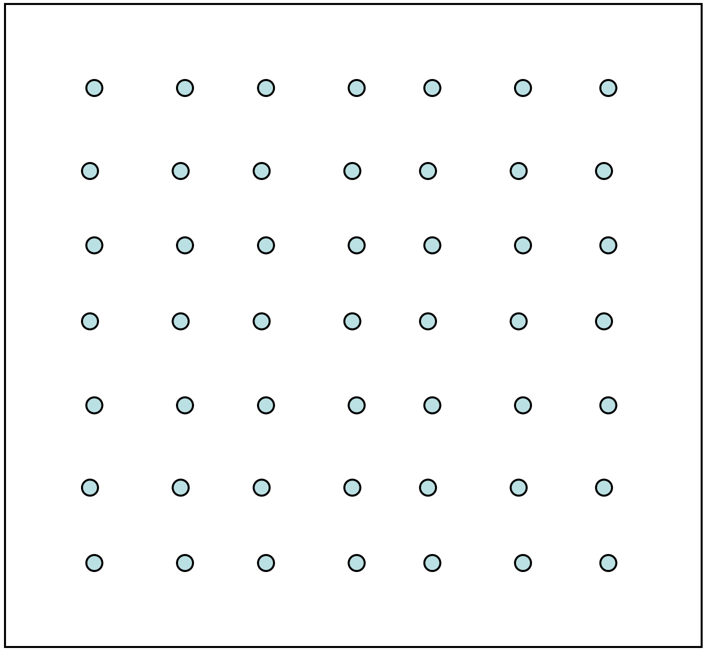
→ node density is n / 1 = n

R transmission range



# Critical Transmission Range

Unit square

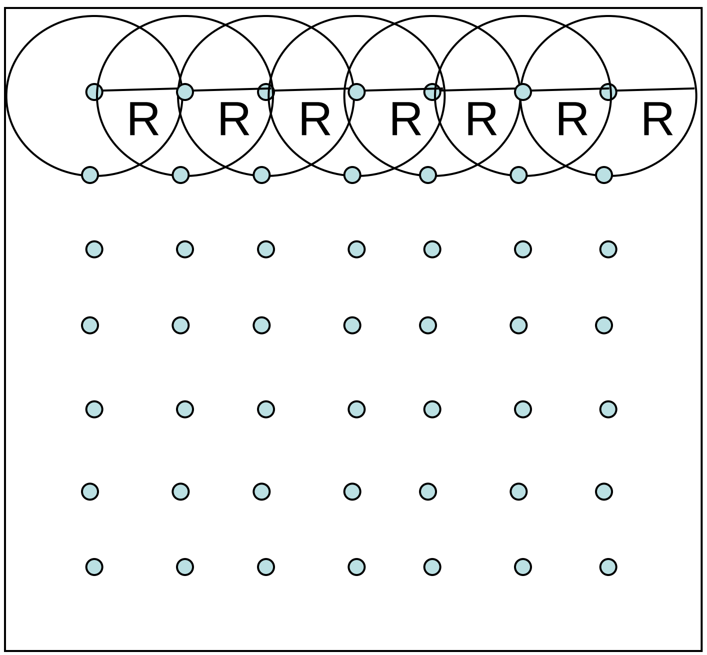


n = 49

Regular grid as reference

# Critical Transmission Range

Unit square



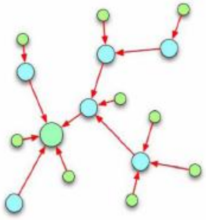

CTR:

R s.t. R [sqrt(n) + 1 ] = 1

→ CTR = 1 / [ sqrt(n) + 1 ]

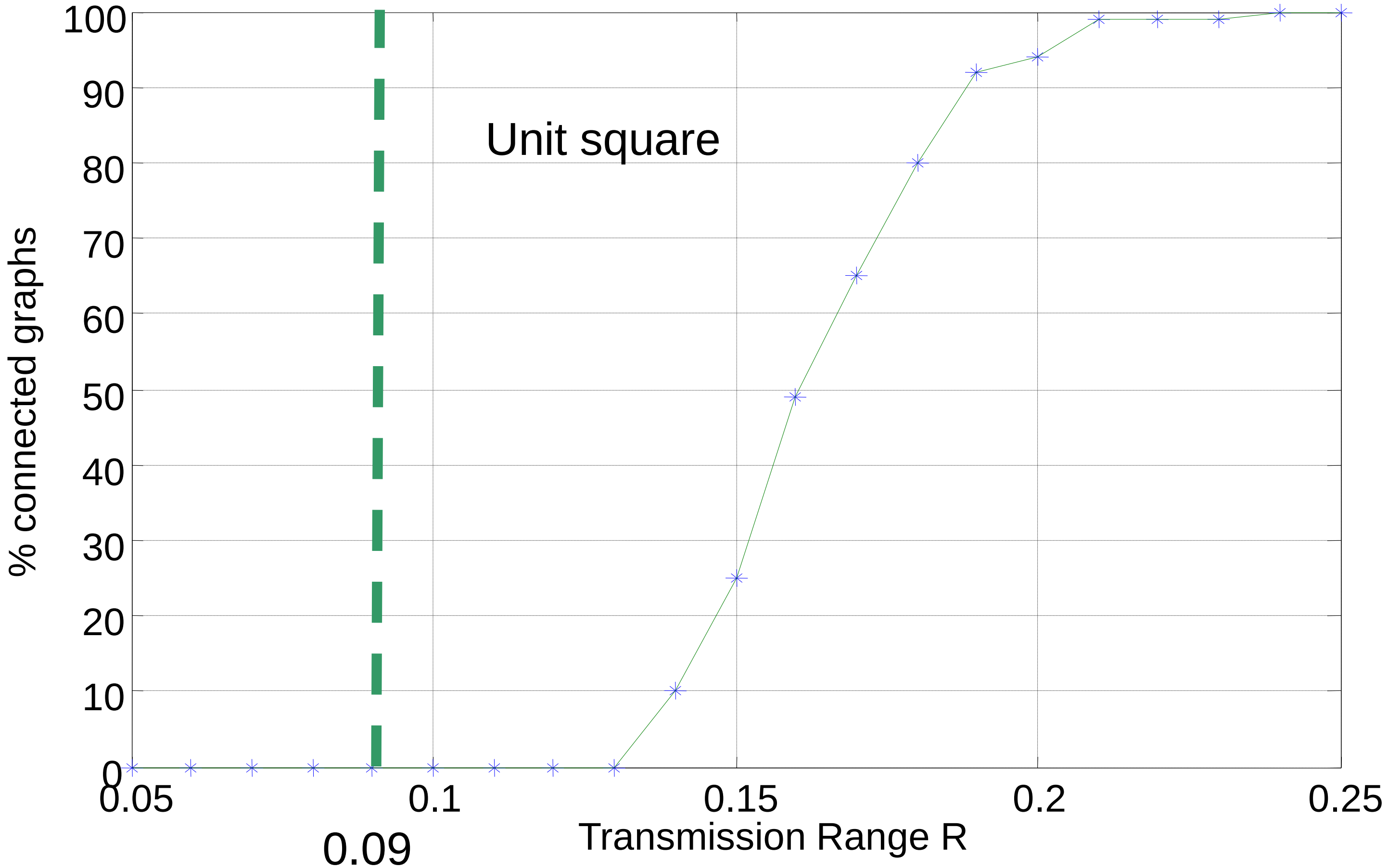
In a square of side L

→ CTR = L / [ sqrt(n) + 1 ]



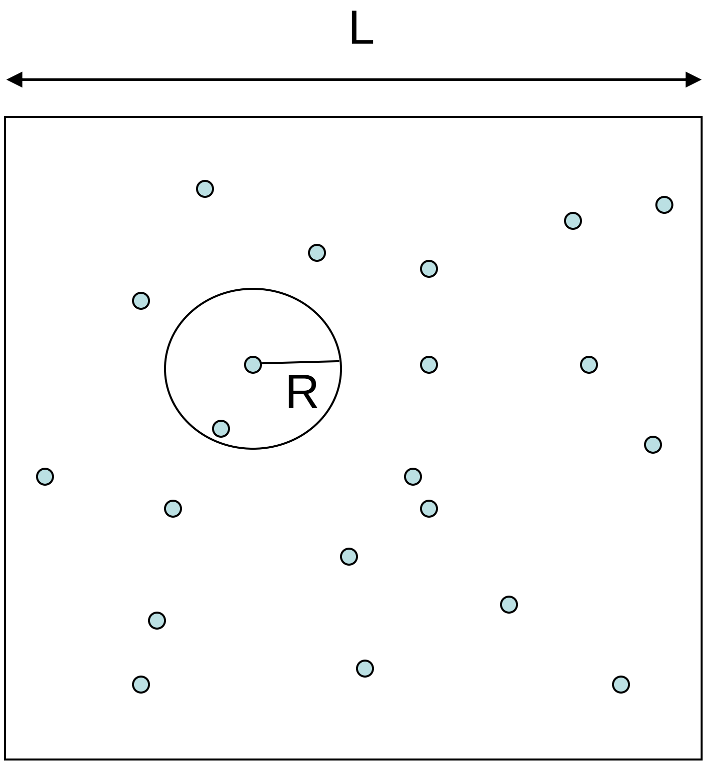
# Critical Transmission Range

n = 100 → CTR = 0.09 in a regular grid 100 graphs




**Randomness of node position has a significant impact on connectivity issues**

# Critical Transmission Range



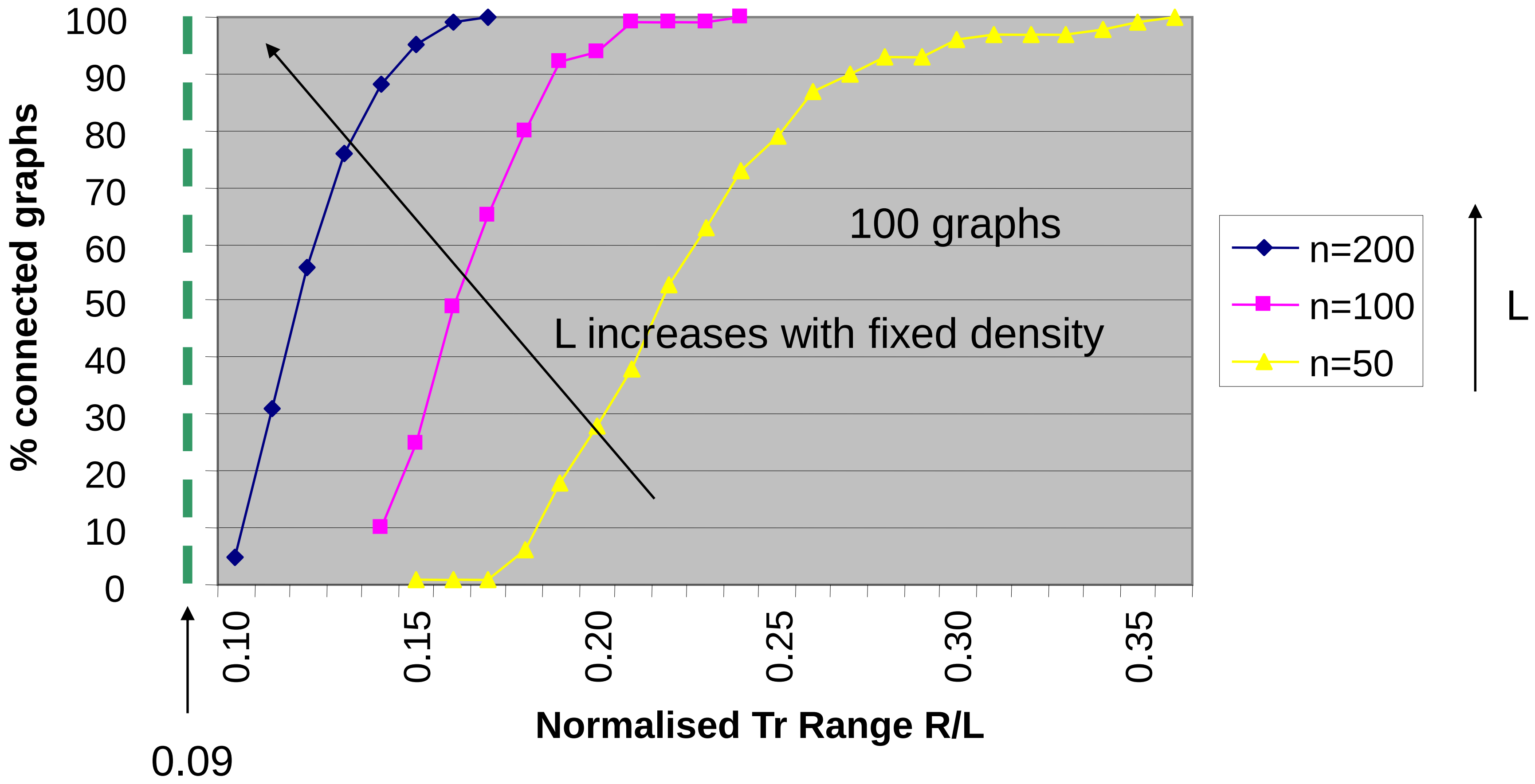

n number of nodes

→ node density is $n / L^2$

→ $n = \rho L^2$

R transmission range

# Critical Transmission Range




**Size of scenario has a significant impact on connectivity issues**

# Critical Transmission Range

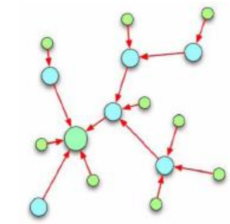
**Theorem (Penrose 1997):**

**Given the unit square and n nodes distributed randomly and uniformly, then the limit for n tending to infinity of**
**Prob [ $n\pi (R_c)^2 - \log n \leq b$]**
**is**
**$1 / \exp(\exp(-b))$**
**for any b in $\mathbf{R}$ where Rc is the CTR.**

As a corollary, for n tending to infinity we have (choose b tending to infinity)
Prob [ $R_c \leq \mathrm{sqrt}( (b + \log n) / n\pi )$] = 1.

In other words, **$\mathrm{sqrt}( (b + \log n) / n\pi )$]** is an upper bound to Rc for n tending to infinity and since it tends to zero for proper selections of b, it is a very tight upper bound.

For finite values of n, this expression does not necessarily represent an upper bound.



# Critical Transmission Range

For which values of n are the predicted values of Rc accurate?

For instance, with b = loglogn

| n | Rc (Penrose) | Rc (sim., 99% conf) | Rc=1 / [ sqrt(n) + 1 ] |
|---|---|---|---|
| 10 | 0.32 | **0.66** | 0.24 |
| 100 | 0.14 | **0,23** | 0.09 |
| 1000 | 0.05 | **0.08** | 0.03 |

Not very accurate for practical values of n.

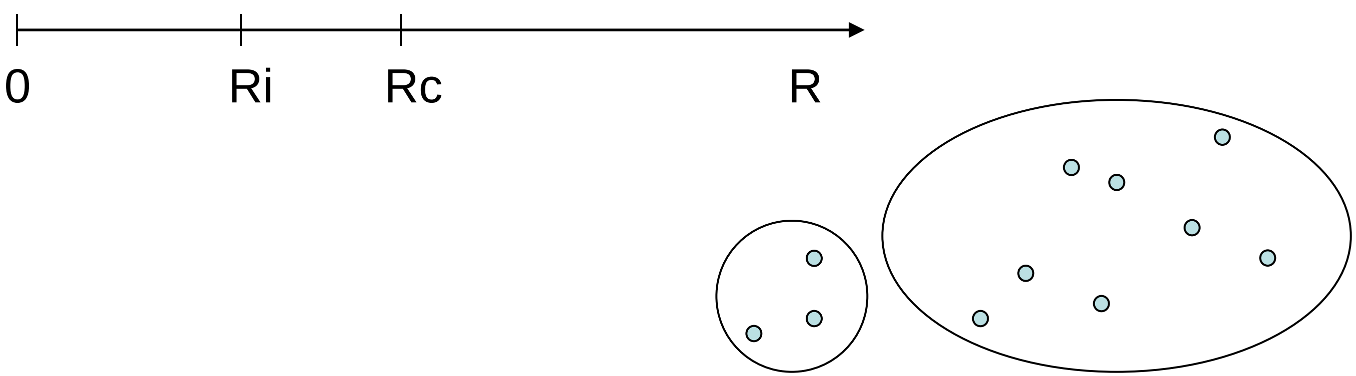
**Considerations for n tending to infinite are often unrealistic for practical values of node densities**

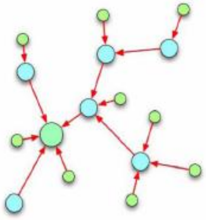
# The Giant Component

Consider a given graph, and let R increase starting from 0.

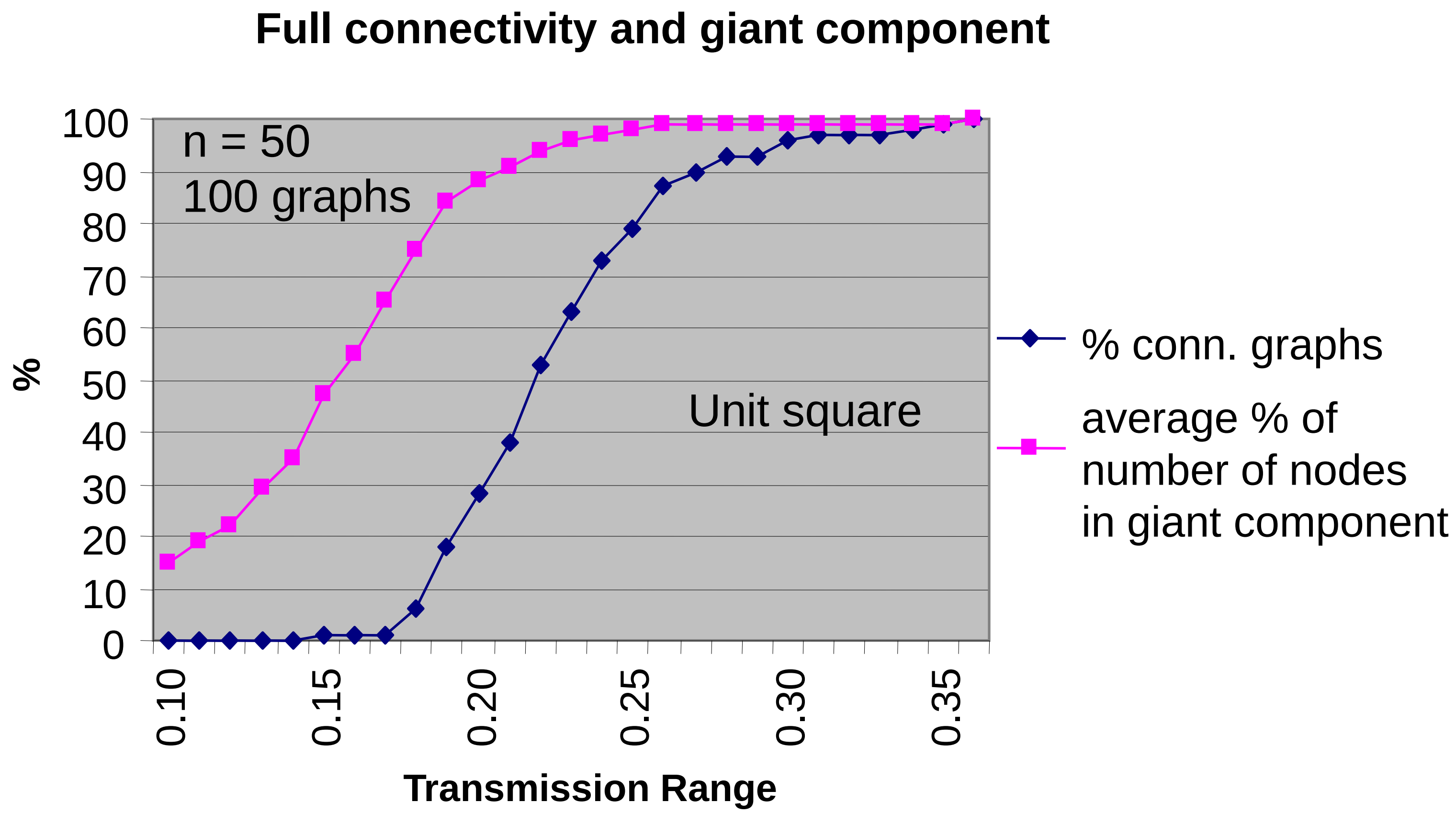
For R = 0 the graph is not connected.
When R increases, nodes group together in clusters.
When R = Ri the last isolated node disappears.
When R = Rc the graph becomes connected.

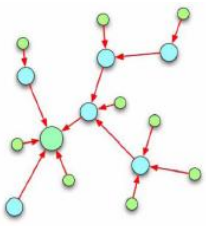
Clearly, Ri < Rc as for some Ri < RA < Rc there might be a communication graph not connected even in the absence of isolated nodes (a “clustered” network with isolated clusters).



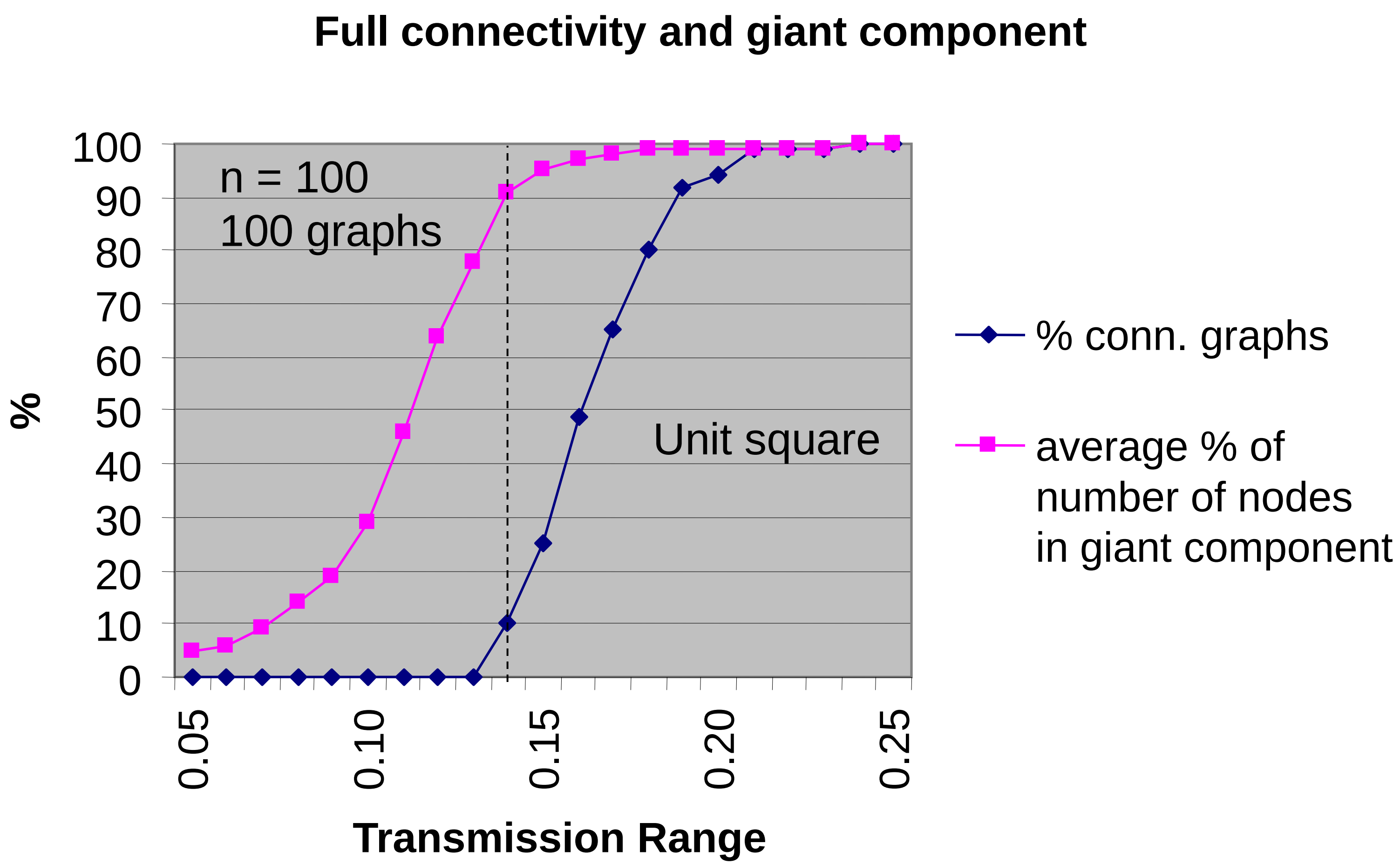


# The Giant Component

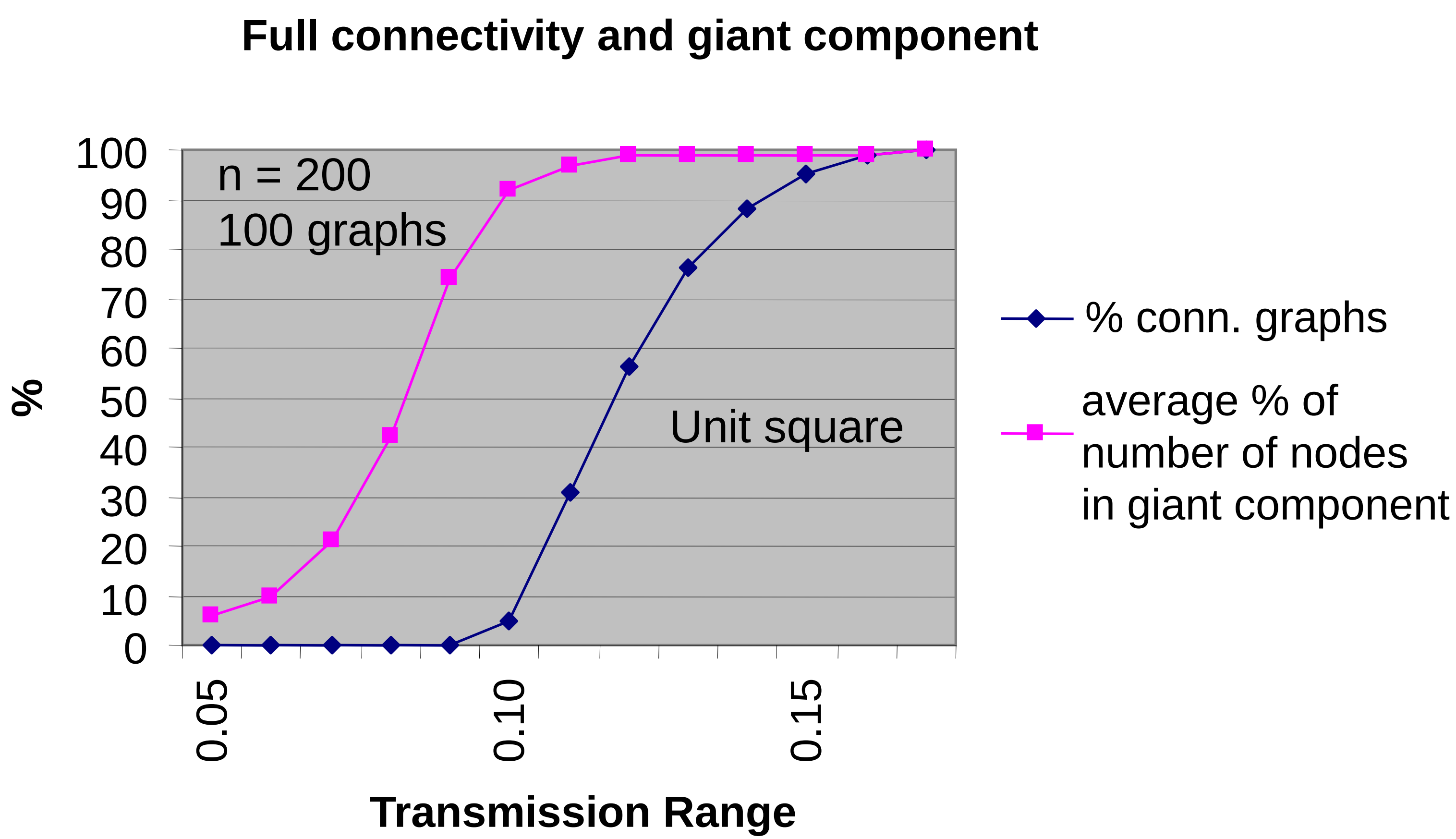




# The Giant Component



# The Giant Component



# The Giant Component

The giant component is a phenomenon appearing also for practical values of n.

Therefore, it can be claimed that, approximately, for any value of n

**the probability of a Communication Graph to be connected equals
the probability of no isolated nodes.**

However, this approximation has not been investigated mathematically for finite n.

# Section 4
# Connectivity Over an Unlimited Region

**John Orriss**
**Orriss' First Result**
**Modification**
**Orriss' Second Result**
**Corollary**

# John Orriss

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 51, NO. 4, APRIL 2003

## Probability Distributions for the Number of Radio Transceivers Which Can Communicate With One Another

[1]

John Orriss and Stephen K. Barton, *Senior Member, IEEE*

PPP over unbounded region
omnidirectional antennas

$$L = k_0 + k_1 \ln R + S \leq l_1.$$

II. Probability Distribution of the Distance Subject to a Maximum Loss

III. Number of Audible Base Stations

# Orriss' First Result

$$f_R(r) = 2Kr\Phi(a - b\ln r) \quad (7)$$

where $a = [k_1 - k_0 - 2\sigma^2/k_1]/\sigma$, $b = k_1/\sigma$, $K = \exp[-(2/k_1)][k_1 - k_0 - \sigma^2/k_1]]$, and $\Phi(x) = \int_{-\infty}^{x}(1/\sqrt{2\pi})e^{-u^2/2}du$.

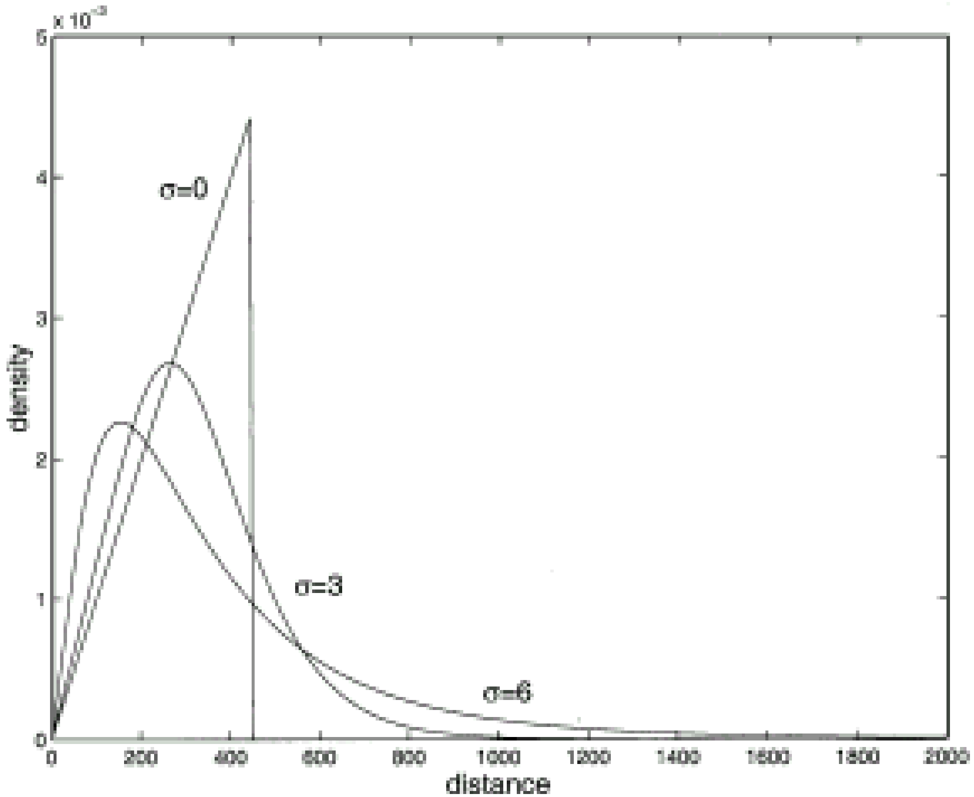



Fig. 1. PDF of distance between communicating nodes.

# Modification

Orriss: $$f_R(r) = 2Kr\Phi(a - b\ln r)$$ where

$$a = [L_{th} - k_0 - 2\sigma^2/k_1]/\sigma,$$

$$b = k_1/\sigma$$

$$K = \exp[-(2/k_1)[L_{th} - k_0 - \sigma^2/k_1]],$$

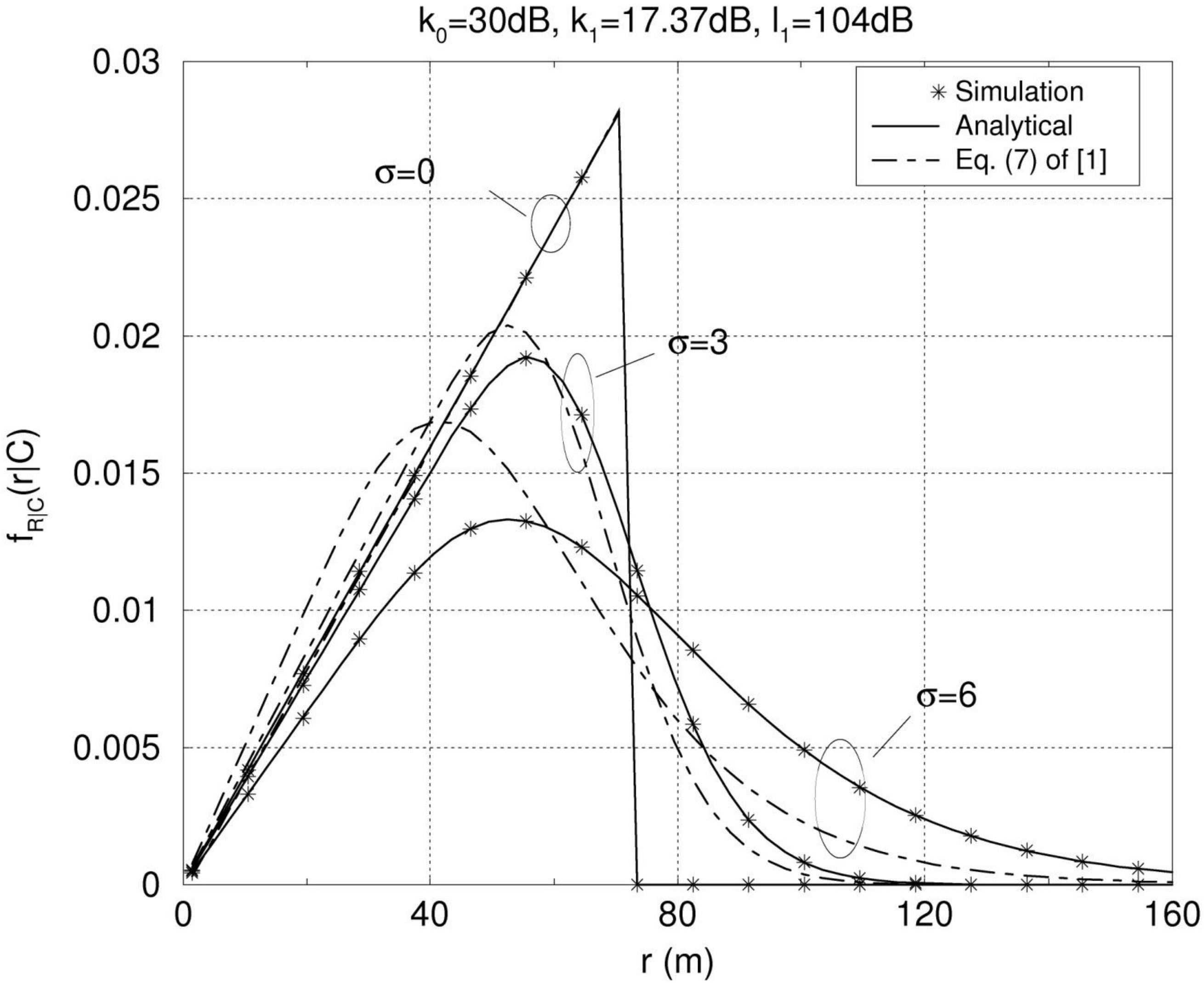
$$\Phi(x) = \int_{-\infty}^{x} (1/\sqrt{2\pi}) e^{-u^2/2} du.$$

Correct expression:

$$f_{R|C}(r|\mathcal{C}) = r\, e^{-\frac{2}{k_1}\left(l_1 - k_0 - \sigma^2/k_1\right)} \mathrm{erfc}\left(\frac{k_0 - l_1 + k_1 \ln r}{\sqrt{2}\sigma}\right)$$

where $\mathrm{erfc}(\cdot)$ denotes the complementary error function.

# Modification

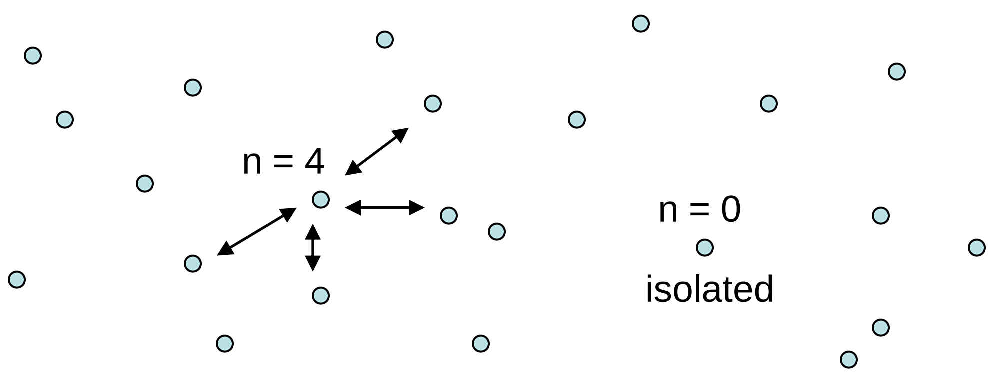


# Orriss' Second Result

The number of nodes heard by a given node is Poisson, with mean

$$Nm = \rho \, \pi \, \exp(2 \, (l_1 - k_0) / k_1) \, \exp(2\sigma^2 / k_1^2)$$

Then, the probability of a node to be isolated is P(iso) = exp (-Nm).



# Orriss' Second Result

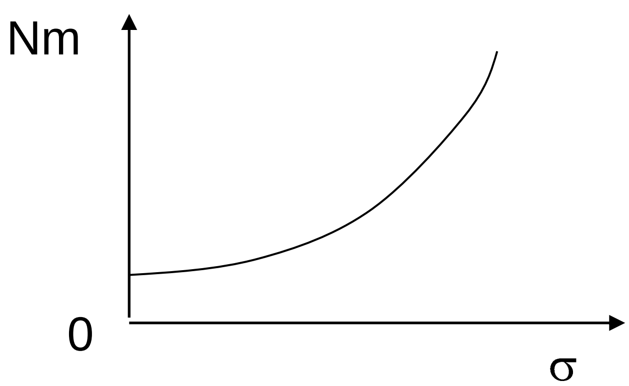
Note: Nm is linearly proportional to node density

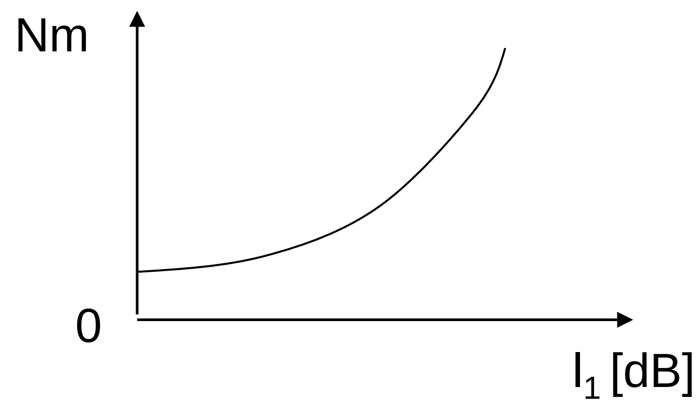
Nm is quadratically proportional to the ideal transmission range R:

$$Nm = \rho \pi R^2 \exp(2\sigma^2 / k_1^2)$$

Nm is exponentially proportional to the variance of shadowing

Nm is exponentially proportional to transmit power: $l_1 = P_t - P_{rmin}$



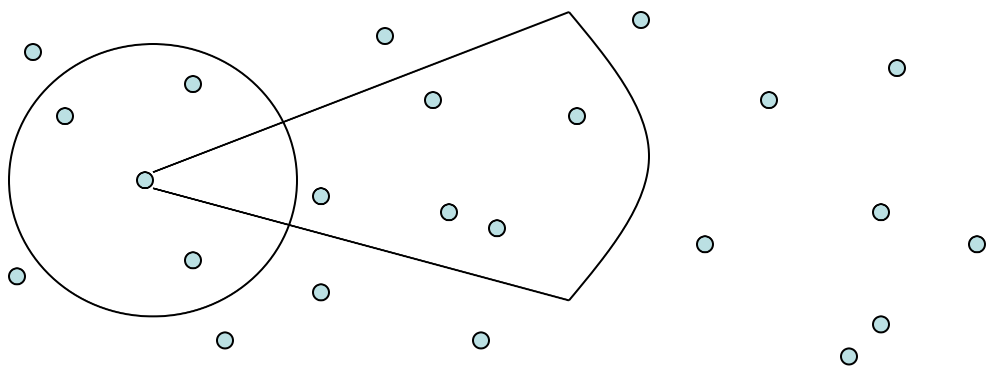

**Channel fluctuations may significantly increase network connectivity**

# Corollary

The number of nodes heard by a given node is Poisson, with mean

$$Nm = \rho \, \pi \, \exp\left(2 \left(l_1 - k_0\right) / k_1\right) \exp\left(2\sigma^2 / k_1^2\right)$$

Do randomly directed directional antennas help increasing connectivity?



# Corollary

The number of nodes heard by a given node is Poisson, with mean

$$Nm = \rho\, \pi \exp\left(2\,(l_1 - k_0) / k_1\right) \exp(2\sigma^2 / k_1^2)$$

**Do randomly directed directional antennas help increasing connectivity?**

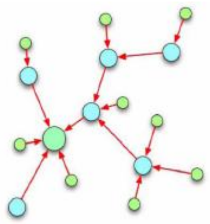
Denoting $Nm = Nm(G)$ the mean as a function of antenna gain, implicitly included in $k_o$, then with directional antennas

$$Nm(G) \quad = (1 / G)\, \rho\, \pi \exp\left(2\,(l_1 - k_0 + 10\log G) / k_1\right) \exp(2\sigma^2 / k_1^2)$$

$$= Nm(1) \exp\left((20\log G) / k_1\right) / G$$

Therefore the ratio $Nm(G) / Nm(1) = \exp\left((20\log G) / k_1\right) / G$ measures the advantage of using directional antennas.

For example:
$G = 4$ [6 dB], $k_1 = 13.3$ → $Nm(G) / Nm(1) = 0.62$ → **connectivity is decreased!**



# Section 5
# Connectivity for WSNs

# Full Connectivity

Traditional definition:

*A network is fully connected if there exists any path (sequence of hops) between every pair of nodes.*

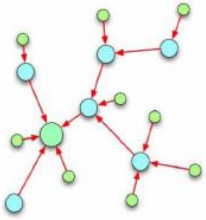
This definition is compliant with the objective of ad hoc networks, i.e. to allow every node being in contact with any other node.

**But this is not the goal of a WSN**.

In WSNs, nodes (sensors) want to transmit their samples to a given node, namely, the sink (or any node in a given set, in the case of multi-sink networks).

Definition more suitable for WSNs:

*A WSN is fully connected if all nodes can report their samples to a sink through any path.*



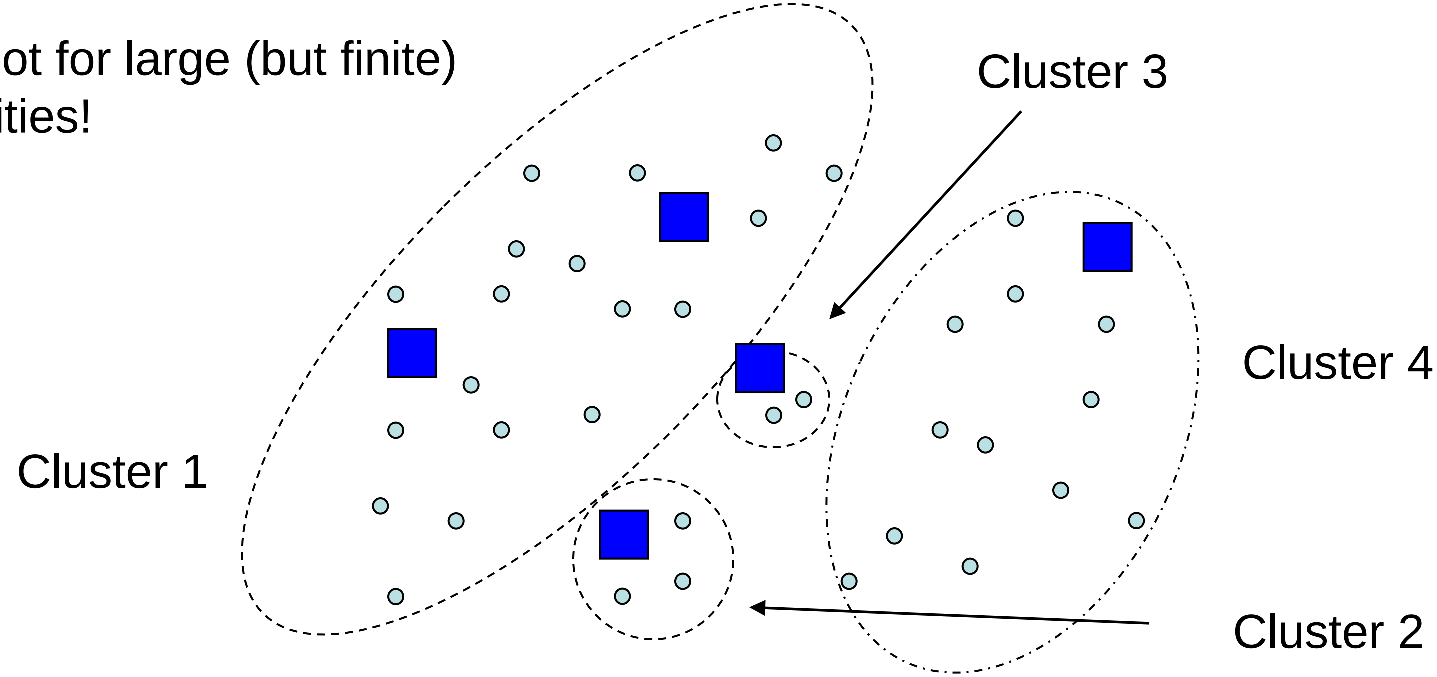
# Full Connectivity

*A WSN is fully connected if all nodes can report their samples to a sink through any path.*

No difference in a single-sink scenario

Difference in a multi-sink scenario

But not for large (but finite) densities!



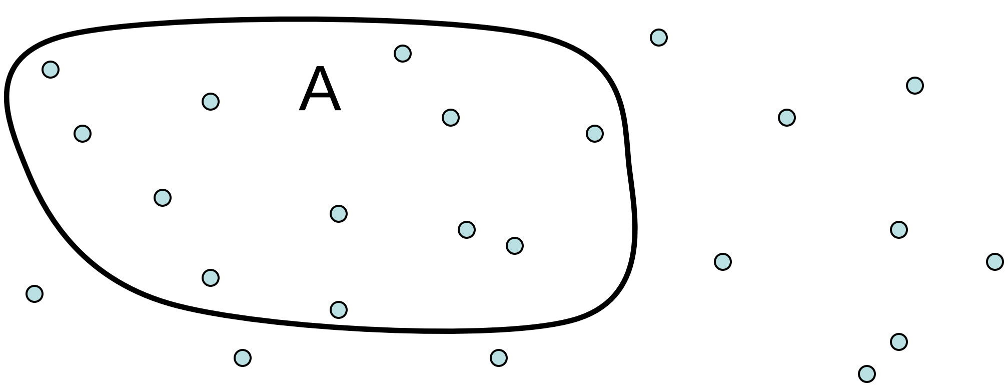
# Full Connectivity: approximated analysis

Probability of a network of area A, given $n_o$ nodes in A, being fully connected:

$$P(con \mid n_o) \underset{(1)}{\approx} P(\text{no isolated nodes in } A \mid n_o) \underset{(2)}{\approx} (1 - P(iso))^{n_o}$$

(1) If network is dense: indeed, $P(con) \leq P(\text{no isolated nodes in A})$
(2) If P(iso) is small and network is dense

| | |
|---|---|
| Dense network: | $\rho A >> 1$ |
| P(iso) small: | $P(iso) << 0.001$ |



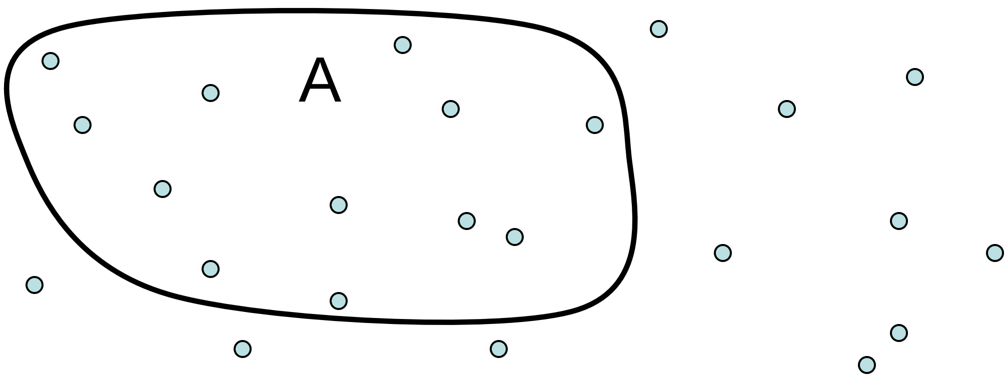
# Full Connectivity: approximated analysis

Probability of a network of area A being fully connected:

$P(con) = E_{n_o}[\, P(con \mid n_o)\,] = E_{n_o}[(1 - P(iso))^{n_o}] = \ldots$

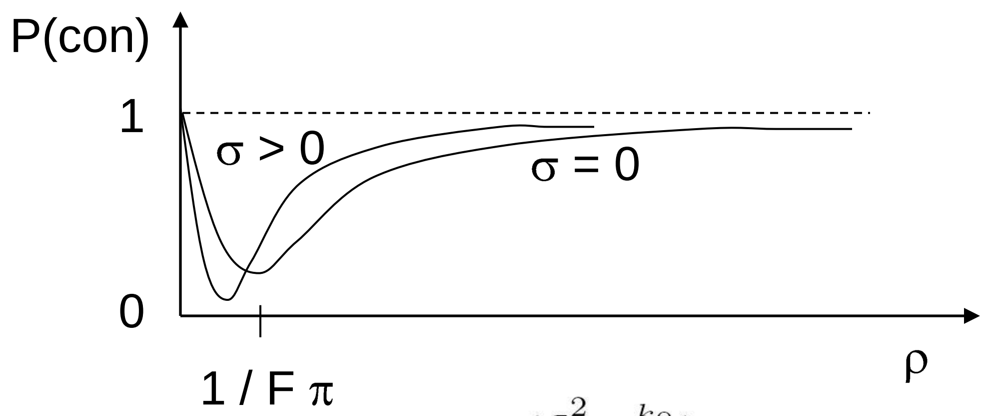
$= \exp[\, -\rho A \exp(-Nm)\,]$ $\qquad Nm = \pi\rho e^{2\left(\frac{\sigma^2}{k_1^2} - \frac{k_0}{k_1}\right)} e^{2\left(\frac{Lth}{k_1}\right)}$ $\qquad [L_{th} = l1]$

Note: A must be large → $A \gg \pi R^2$



# Full Connectivity: approximated analysis

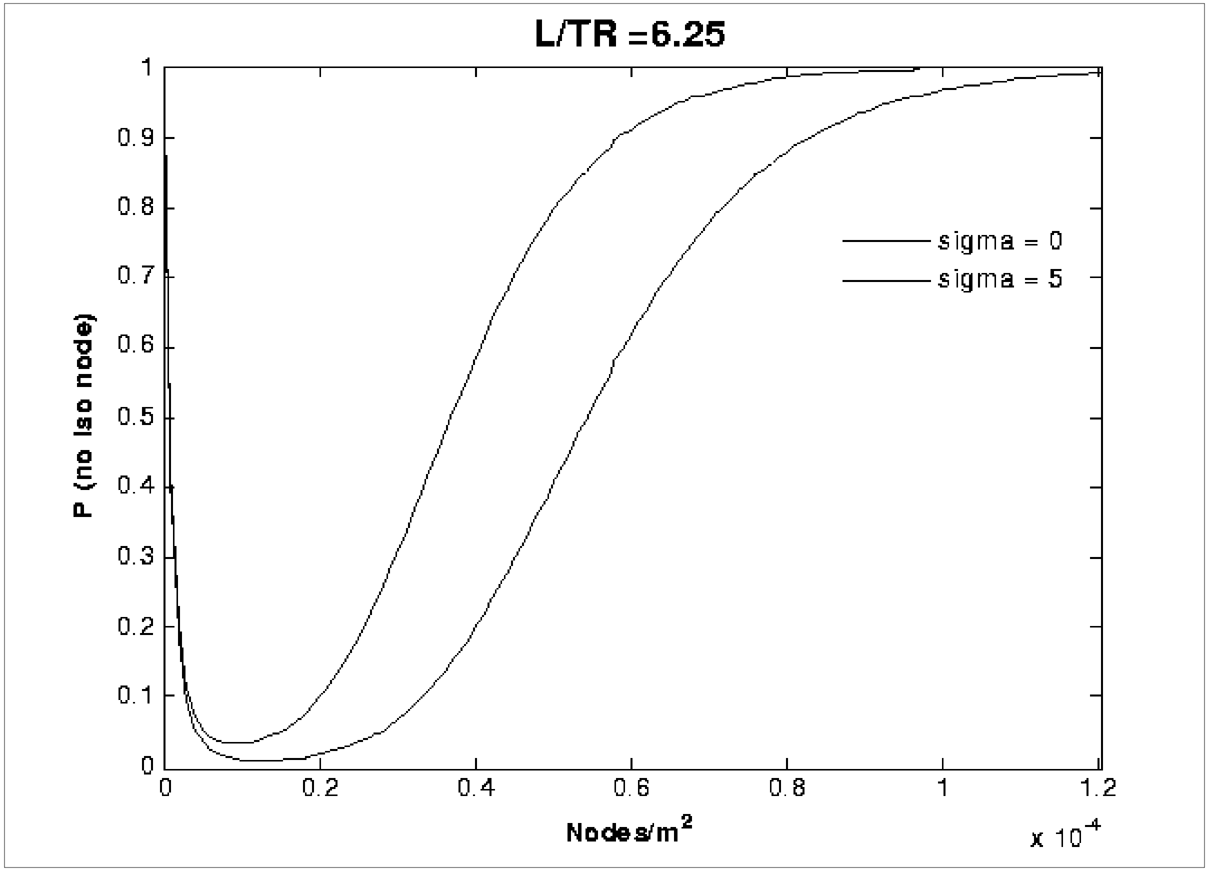
Probability of a network of area $A = L^2$ being fully connected




$$F = e^{2\left(\frac{\sigma^2}{k_1^2} - \frac{k_0}{k_1}\right)} e^{2\left(\frac{Lth}{k_1}\right)}$$

# Full connectivity: approximated analysis

Probability of a network of area $A = L^2$ being fully connected. TR is ideal trans. range

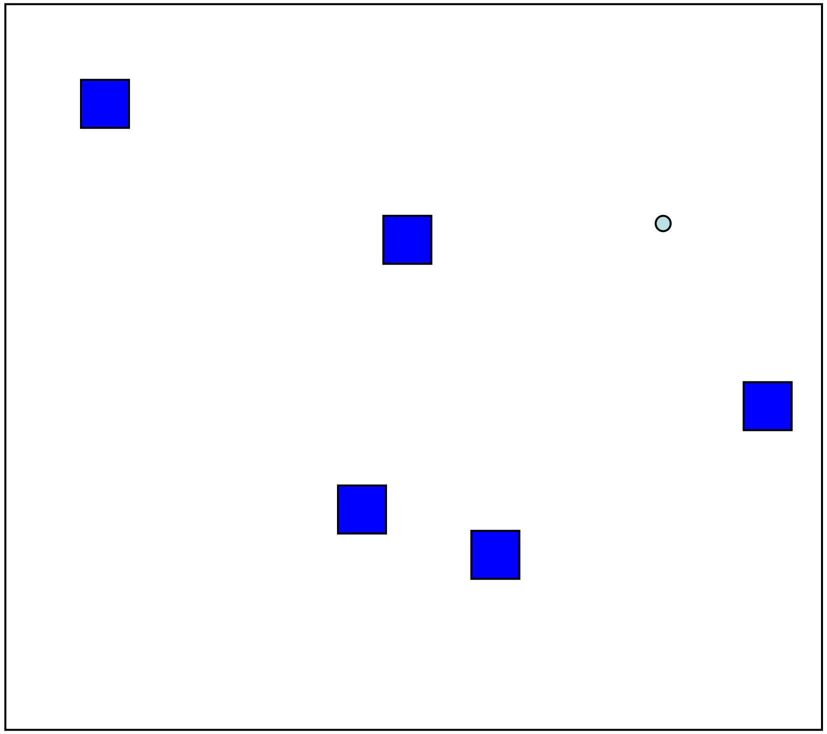



**Channel fluctuations significantly reduce the number of required nodes**

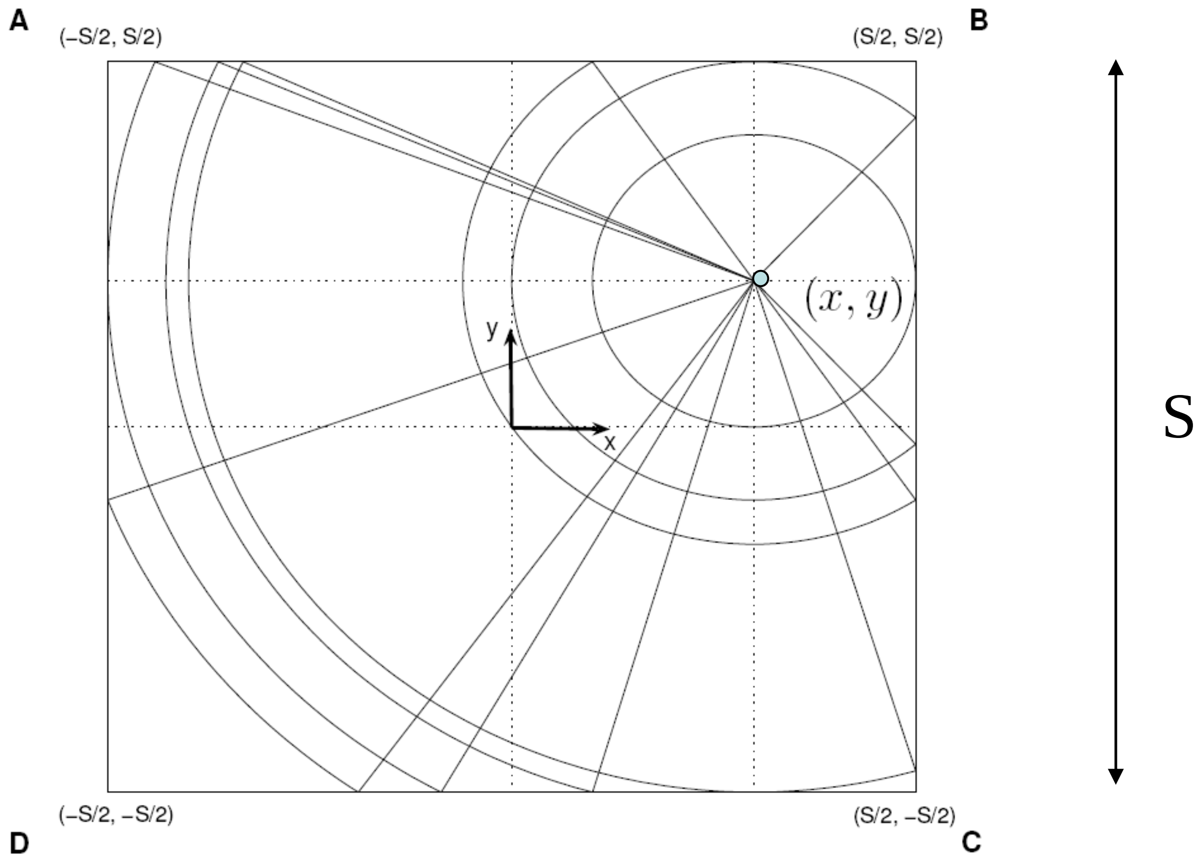
# Section 6
# Connectivity Over Limited Regions for WSNs

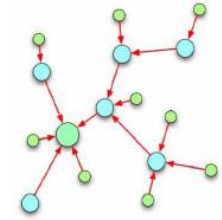
## Connectivity in Squares
## Full Connectivity in Squares – Single Hop
## Reachability in Squares – Single Hop
## Reachability in Squares – Multiple Hops, Tree Topologies

# Connectivity in Squares



# Connectivity in Squares





# Connectivity in Squares

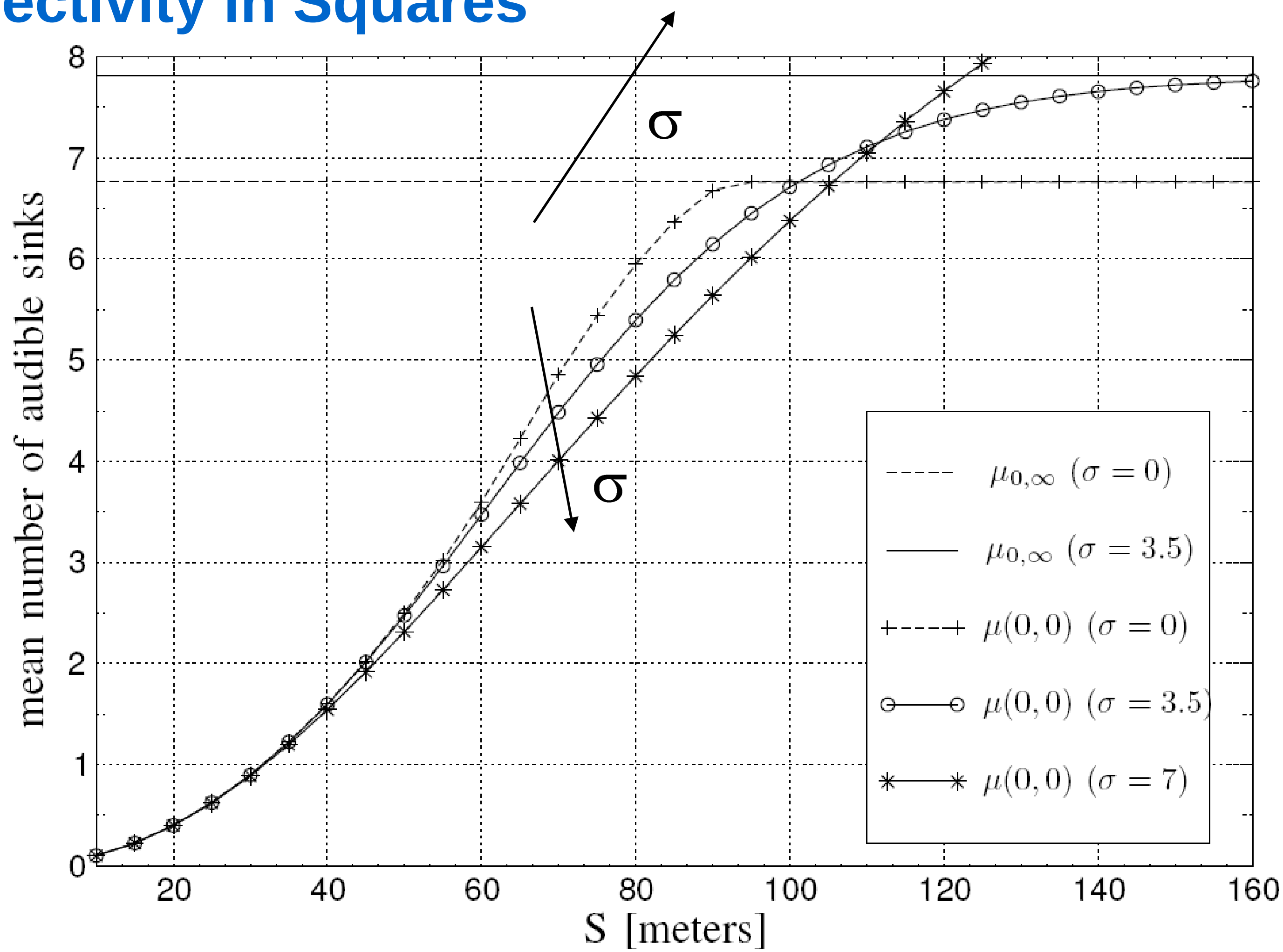
Number of sinks heard from a node in (x,y) is Poisson with mean

$$\mu(x,y)=\sum_{i=1}^{8}\int_{r_{1,i}}^{r_{2,i}} 2\theta_i(r)\cdot\rho_0\cdot r\cdot\Phi(a_1-b_1\ln r)dr$$

| Region | Range: $r_1 \le r \le r_2$ |
|---|---|
| 1 | $0\le r\le \frac{S}{2}-x$ |
| 2 | $\frac{S}{2}-x\le r\le \frac{S}{2}-y$ |
| 3 | $\frac{S}{2}-y\le r\le\sqrt{(\frac{S}{2}-x)^2+(\frac{S}{2}-y)^2}$ |
| 4 | $\sqrt{(\frac{S}{2}-x)^2+(\frac{S}{2}-y)^2}\le r\le \frac{S}{2}+y$ |
| 5 | $\frac{S}{2}+y\le r\le\sqrt{(\frac{S}{2}-x)^2+(\frac{S}{2}+y)^2}$ |
| 6 | $\sqrt{(\frac{S}{2}-x)^2+(\frac{S}{2}+y)^2}\le r\le\frac{S}{2}+x$ |
| 7 | $\frac{S}{2}+x\le r\le\sqrt{(\frac{S}{2}+x)^2+(\frac{S}{2}-y)^2}$ |
| 8 | $\sqrt{(\frac{S}{2}+x)^2+(\frac{S}{2}-y)^2}\le r\le\sqrt{(\frac{S}{2}+x)^2+(\frac{S}{2}+y)^2}$ |

| Region | $\theta(r)$ |
|---|---|
| 1 | $\pi$ |
| 2 | $\frac{\pi}{2}+\arcsin\frac{\frac{S}{2}-x}{r}$ |
| 3 | $\frac{\pi}{2}+\arcsin\frac{\frac{S}{2}-x}{r}-\arccos\frac{\frac{S}{2}-y}{r}$ |
| 4 | $\frac{\pi}{2}+\frac{1}{2}(\arcsin\frac{\frac{S}{2}-x}{r}-\arccos\frac{\frac{S}{2}-y}{r})$ |
| 5 | $\frac{\pi}{2}-\arccos\frac{\frac{S}{2}+y}{r}+\frac{1}{2}(\arcsin\frac{\frac{S}{2}-x}{r}-\arccos\frac{\frac{S}{2}-y}{r})$ |
| 6 | $\frac{\pi}{2}-\frac{1}{2}(\arccos\frac{\frac{S}{2}+y}{r}+\arccos\frac{\frac{S}{2}-y}{r})$ |
| 7 | $\frac{1}{2}(\arcsin\frac{\frac{S}{2}-y}{r}+\arcsin\frac{\frac{S}{2}+y}{r})-\arccos\frac{\frac{S}{2}+x}{r}$ |
| 8 | $\frac{1}{2}(\arcsin\frac{\frac{S}{2}+y}{r}-\arccos\frac{\frac{S}{2}+x}{r})$ |

# Connectivity in Squares




**Channel fluctuations increase network connectivity only in unlimited region**

# Full Connectivity in Squares – Single Hop

$$q(x,y) = 1 - \exp(-\mu(x,y))$$

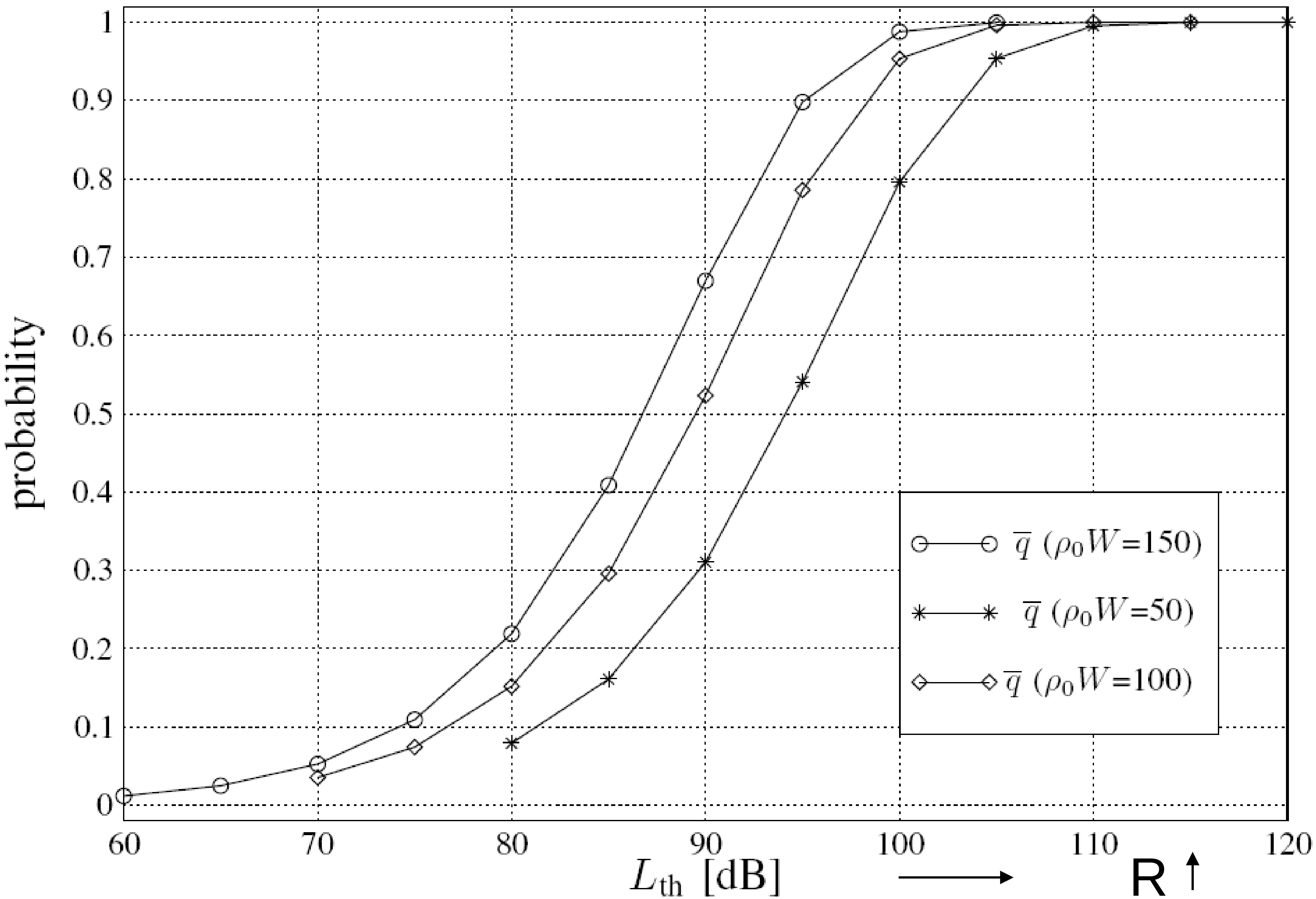
$$\overline{q} = \frac{8}{W} \int_0^{S/2} \int_0^{x} q(x,y)\,dy\,dx$$

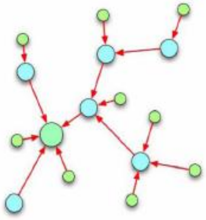
$$\text{Prob}\{F | N_{\text{s}} = k\} = \bar{q}^k$$

$$Z = \text{Prob}\{F\} = \sum_{k=1}^{+\infty} \text{Prob}\{F | N_{\text{s}} = k\}\text{Prob}\{N_{\text{s}} = k\}$$

$$= \sum_{k=1}^{+\infty} \bar{q}^k \cdot \frac{e^{-\rho_s W}}{k!} (\rho_s W)^k$$

# Full Connectivity in Squares – Single Hop





# Reachability in Squares – Single Hop

In some applications, Full Connectivity is not required

A sufficient degree or reachability is requested i.e. a given minimum number of sensors needs to reach the sinks

$$Z_{\bar{m}}(j) = \mathrm{Prob}\left\{C_j\right\} = e^{-\bar{m}} \cdot \sum_{k=j}^{+\infty} \sum_{l=j}^{k} \binom{k}{l} \frac{\bar{m}^k \bar{q}^l (1-\bar{q})^{k-l}}{k!}$$

# Reachability in Squares – Single Hop