Sistemi Radio M
Fundamentals of Digital Transmission

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Credits: 6
The scope of this lecture block is to introduce the concept of Capture Effect
Capture Effect

SNR = \frac{P_r}{P_n}

SIR = \frac{P_r}{P_i}

BER [\log]

BLER [\log]
1. System Model
System Model

Demodulation can be

Linear (normally used for linear modulated signals like BPSK, M-QAM, MSK)
Non Linear (normally used for phase or frequency modulated signals like GMSK)

\[
G = \frac{1}{A} = |Fp(f)|^2
\]

Bit rate \( R_b = \frac{1}{T_b} \)
Symbol rate \( R_s = \frac{1}{T} \)
\[
\{a_n\} \text{ in } \{\pm (L-1); \ldots; \pm 3; \pm 1\} \quad L = 2, 4, 8, 16, \ldots
System Model
2. Link Performance over AWGN
Link Performance over AWGN

\[ \text{SNR} = \frac{\text{Pr}}{\text{Pn}} \]

\[ s(t) = s_t(t) + \nu(t) \]

\[ s_r(t) = \text{BER} \]

\[ \text{Pn} = \text{No Bc} \]

noise spectral density

frequency

No
Link Performance over AWGN

\[
\text{SNR} = \frac{\text{Pr}}{\text{Pn}}
\]

Gaussian

Linear Demodulation

pdf
Link Performance over AWGN

\[ \text{BER} = \frac{\text{Pr}}{\text{Pn}} \]

\[ \text{SNR} = \frac{\text{Pr}}{\text{Pn}} \]

noise spectral density

Non Linear Demodulation

Gaussian
Link Performance over AWGN

\[ \text{SNR} = \frac{\text{Pr}}{\text{Pn}} \]

BER

Complexity

SNR [dB]
Link Performance over AWGN

\[ \text{SNR} = \frac{\text{Pr}}{\text{Pn}} \]

We will assume it is known

Reference Systems:

- Non coherent BPSK: \( \text{BER} = 0.5 \exp(-\text{SNR}/2) \)
- Coherent BPSK: \( \text{BER} = 0.5 \ \text{erfc}(\sqrt{\text{SNR}}) \)
- Coherent QPSK: \( \text{BER} = 0.5 \ \text{erfc}(\sqrt{\text{SNR}}) \)
- Coherent M-QASK: \( \text{BER} = \left[\frac{(L-1)}{(L \log L)}\right] \text{erfc}(\sqrt{\text{SNR} \log L / (L-1)^2}) \)  \( L = \sqrt{M} \)
Link Performance over AWGN

Example: Mathematical Derivation
3. Link Performance over AWGN with Interference
Link Performance over AWGN with Interf.

\[ \text{BER} = \log \left( \text{SNR} \right) \]

\[ \text{SIR} = \frac{\text{Pr}}{\text{Pri}} \]

\[ s_r(t) = s_{ti}(t) + v(t) \]

\[ S_r(t) = s_{ri}(t) \]

\[ P_t \]

\[ P_{ti} \]
Link Performance over AWGN with Interf.

Noise neglected

\[ S_{ri}(t) = \frac{Pr}{Pri} \]

\[ s_{ri}(t) \]

\[ s_{i}(t) \]

\[ d_{i}(t) \]

\[ \text{pdf} \]
Link Performance over AWGN with Interf.

\[
S_R(t) = \frac{P_R}{P_I}
\]

Noise neglected

Non Linear Demodulation

pdf
Link Performance over AWGN with Interf.

\[ \text{SIR} = \frac{P_r}{P_i} \]

Noise neglected

Non Linear Demodulation

\[ \text{BER} \text{ [log]} \]

\[ \text{SNR} \text{ [dB]} \]

\[ \text{BER} \text{ [log]} \]

never zero
Link Performance over AWGN with Interf.

\[ S_i(t) \rightarrow T \rightarrow \frac{P_t}{P_i} \]

\[ S_r(t) \rightarrow R \rightarrow \text{BER} \]

\[ S_r(t) = S_i(t) + S_i(t) \]

\[ \text{SIR} = \frac{P_r}{P_i} \]

Noise neglected

\[ s_{ri}(t) \rightarrow RA \rightarrow D \rightarrow \text{Det} \]

Linear Demodulation

\[ s_{ri}(t), s_i(t), d_i(t) \]

pdf

BER [log]

SNR [dB]
Link Performance over AWGN with Interf.

\[ S_{ri}(t) = \frac{P_r}{P_{ri}} \]

Noise neglected

Linear Demodulation
Link Performance over AWGN with Interf.

Noise included

\[
\text{SIR} = \frac{\text{Pr}}{\text{Pri}}
\]

\[
\nu(t) + s(t)
\]

\[
\text{Pt}
\]

\[
\text{Pti}
\]

BER [log]

SNR [dB]

pdf
Link Performance over AWGN with Interf.

Noise limited system

\[ C = Bc \log_2 (1 + SNR) \]

\[ C = Bc \log_2 (1 + SIR) \]

Interference limited system

\[ \text{SINR} = \frac{P_r}{P_n + P_{ri}} \]
Link Performance over AWGN with Interf.

Interference limited system

BPSK, matched filter detection

Analytical Distribution of Linearly Modulated Cochannel Interferers

Marco Chiani, Member, IEEE

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 45, NO. 1, JANUARY 1997
4. Link Spectrum Efficiency
**Link Spectrum Efficiency**

Bit rate transmitted per spectrum unit

\[ \eta = \frac{R_b}{B_c} \quad \text{[bit/s/Hz]} \]

\[ \eta < \frac{C}{B_c} = \log_2 [1 + \text{SNR}] \]

It measures spectrum compactness
5. Link Spectrum Efficacy
Link Spectrum Efficacy:
Throughput transmitted per spectrum unit under interference lim. conditions

\[ \eta_{\text{eff}} = \frac{Th}{Bc} \quad \text{[bit/s/Hz]} \quad \text{Th} = \text{throughput @ given SIR} \]

\[ \eta_{\text{eff}} < \frac{C}{Bc} = \log_2 [1 + \text{SIR}] \]

It measures spectrum efficiency

Be careful with link spectrum efficiency values!
6. FEC, ARQ
FEC, ARQ

FEC: Forward Error Correction

k bit data block

next block

n bit data block

next block

up to t errors corrected

time at source

time at sink

BLER

BER [log]

SNR [dB]

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FEC, ARQ

ARQ: Automatic Repeat Request

Stop and Wait

k bit data block

n bit data block

next block

acknowledgement (ack)

errors detected

data block

time

BER [log]

SNR [dB]
FEC, ARQ

ARQ: Automatic Repeat Request

Go Back N

k bit data block | next block | next block

n bit data block

errors detected

acknowledgement (ack)

time at source

time at sink
FEC, ARQ

FEC: Forward Error Correction

\[ \text{BLER} = f(\text{BER, n, t}) \]

BLER is a function of BER, n, and t.

ARQ: Automatic Repeat Request

\[ \text{BLER} = 1 - (1 - \text{BER})^n \]

BLER increases as t increases.

\[ \text{BLER} \approx N \times \text{BER} \]

BLER is approximately equal to N times BER at high SNR.
FEC, ARQ

Blind Error Rate (BLER) and Signal to Noise Ratio (SNR) are key metrics in wireless communication systems. The diagram illustrates the relationship between BLER, SNR, and receiver sensitivity (Pr* / Pn). BLER is often plotted on a logarithmic scale (BLER [log]) against SNR [dB].

Pr* is denoted as receiver sensitivity.
Example: BCH FEC over QPSK

Mathematical Derivation

![Graph showing BLER vs SNR for different coders]

- Coderate = 1/2
- Coderate = 7/8
- Coderate = 1

n = 512
Homework

A radio system uses QPSK over an AWGN channel with ARQ. Data blocks have size 1 Kbytes. The channel bandwidth is 22 MHz, centred at carrier frequency 2.45 GHz. The (monolateral) noise density power is $10^{-20}$ W/Hz. Determine the receiver sensitivity [dBm] defined as the minimum received power ensuring BLER=0.01. Does it depend on the data block size?

The same system uses BCH FEC (with n = 512 and coderate 7/8) instead of ARQ. Determine the receiver sensitivity [dBm] defined as the minimum received power ensuring BLER=0.01. Does it depend on the data block size?
7. Capture Effect
Capture Effect

\[ \text{SNR} = \frac{P_r}{P_n} \]
\[ \text{SIR} = \frac{P_r}{P_i} \]

Block Error Rate (BLER) [log] vs. Signal-to-Interference Ratio (SIR) [dB]

useful data block

interfering data block

data blocks transmitted simultaneously
Capture Effect

SNR = \frac{Pr}{Pn}

SIR = \frac{Pr}{Pri}

BLER [\log]

protection ratio $\alpha$

data blocks transmitted simultaneously

useful data block

interfering data block

time
Capture Effect

SNR = \frac{Pr}{Pn}
SIR = \frac{Pr}{Pri}

If SIR > \alpha then data block is captured by receiver, otherwise lost
Capture Probability: \text{Pc} = \text{Prob} [ \text{SIR} > \alpha ]
Capture Effect

\[
\text{SNR} = \frac{P_r}{P_n}
\]

\[
\text{SIR} = \frac{P_r}{P_{ri}}
\]

Infinite SIR

Useful data block

Interfering data block

Data blocks transmitted simultaneously partially overlapped

The minimum level of SIR is considered
Capture Effect

With multiple interferers, the interference power is the sum of powers.

If $\text{SIR} > \alpha$ then data block is captured by receiver, otherwise lost.

Capture Probability: $P_c = \text{Prob} [ \text{SIR} > \alpha ]$; $\alpha$ can be a function of $n$.
Capture Effect

Example: 802.15.4 Measurements

1 - BLER

\[ SIR [\text{dB}] \]

\[ \alpha \]
Inquiry Based Session

Assume multiple nodes have to transmit to a given sink node. Positions are known, received powers are known. How can we ensure capture if the nodes need to transmit simultaneously towards a given receiver? (make the sink node as complex as you want)
Inquiry Based Session

What is the underlying consideration behind the assumption that the overall interference power is the sum of powers?

Does the consideration always hold?

\[
SIR = \frac{P_r}{P_i} \\
P_i = \text{Sum Pri}
\]
Fundamentals of Digital Transmission

The End