RRC

Mobile Radio Networks

Radio Resource Assignment in Cellular Networks

Roberto Verdone
www.robertoverdone.org

Outline

1. **Cellular Networks**
2. **Reuse**
3. **Network Spectrum Efficiency**
4. **Cluster Size Dimensioning**
5. **Exercises**

The scope of this lecture block is to introduce the basic concepts related to the use of the radio spectrum as a shared and limited resource in cellular networks.

1. Cellular Networks

Area Coverage: Hot Spot

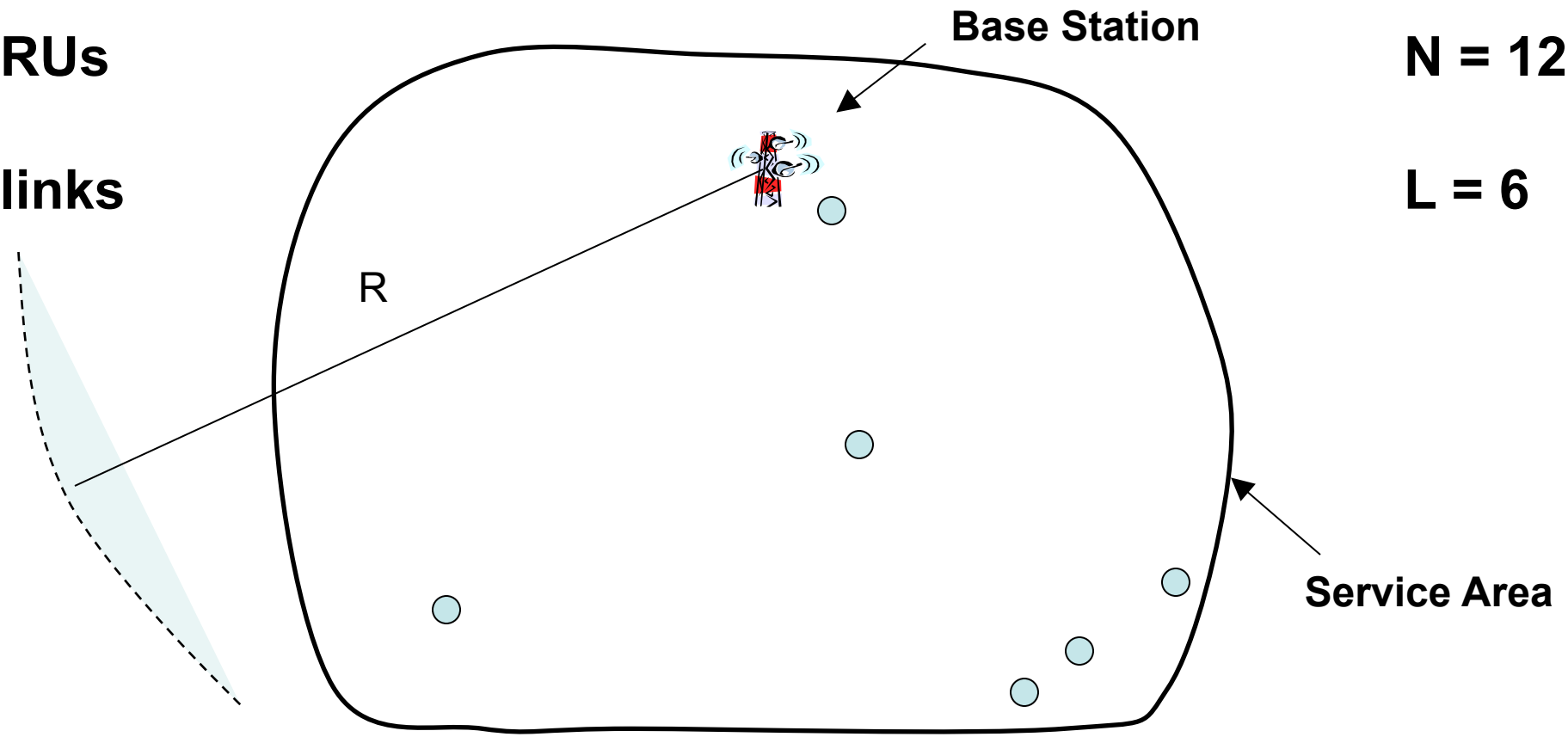
Ex.:

N = 12

L = 6

N RUs

L links



Assumptions: N > L R sufficiently large

Area Coverage: Hot Spot

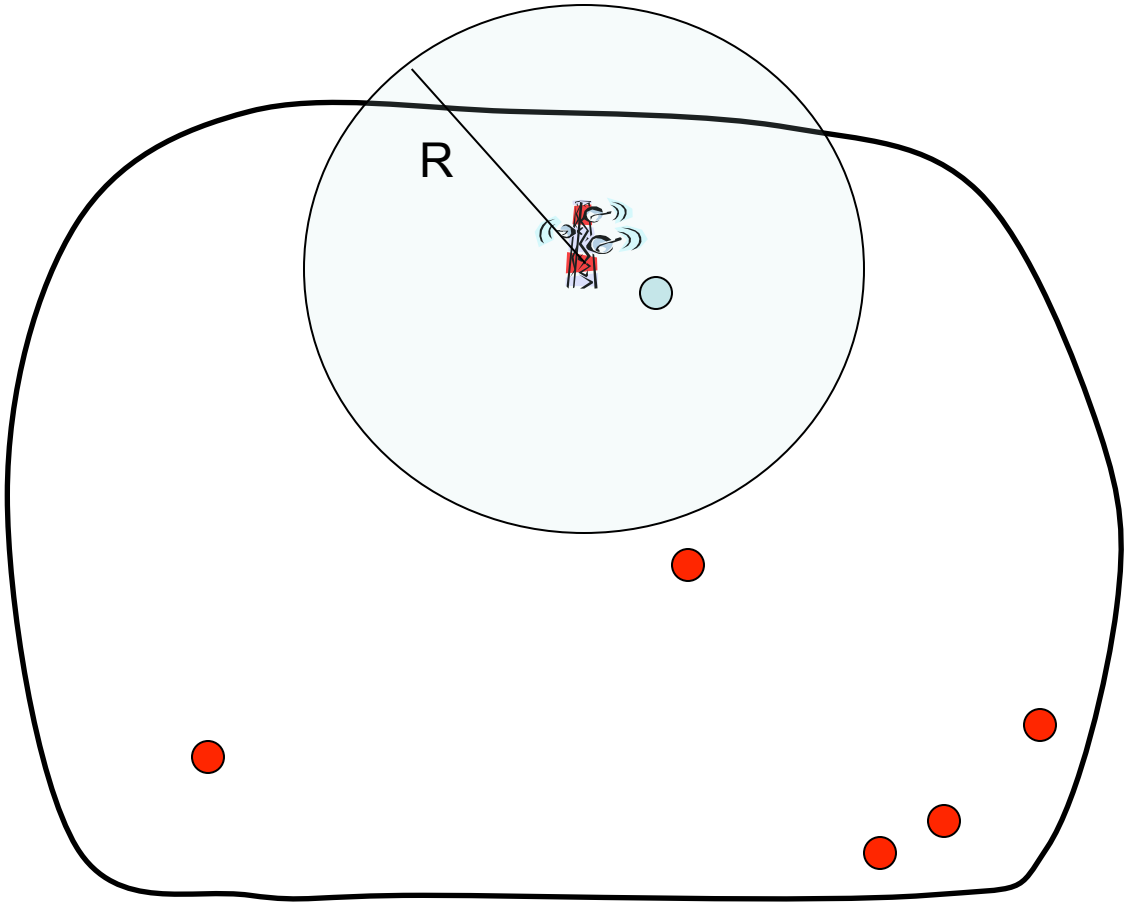
N RUs

L links

Ex.:

N = 12

L = 6



Assumptions: N > L R not sufficiently large

Area Coverage: Procedure

1) Coverage Based Cell Dimensioning ($R < R_{cov_max}$)

Area Coverage: Procedure

1) **Coverage Based Cell Dimensioning ($R < R_{cov_max}$)**

2) **Identify Number of Cells** → **Cost of Deployment**

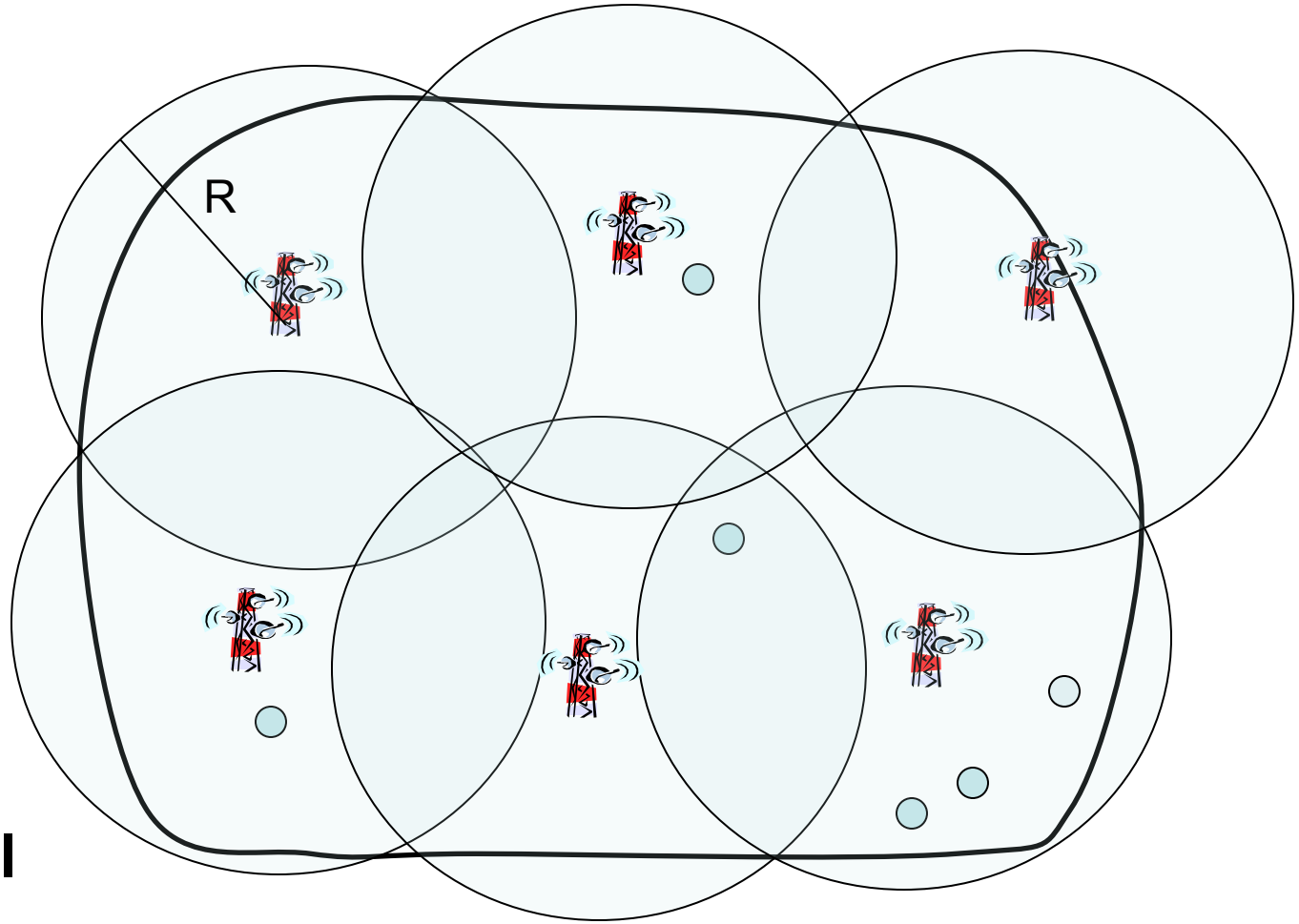
Area Coverage: Cells

Ex.:

N RUs — **N = 12**

L links — **L = 6**

Z cells — **Z = 6**



n RUs/cell

n = N / Z — **n = 2**

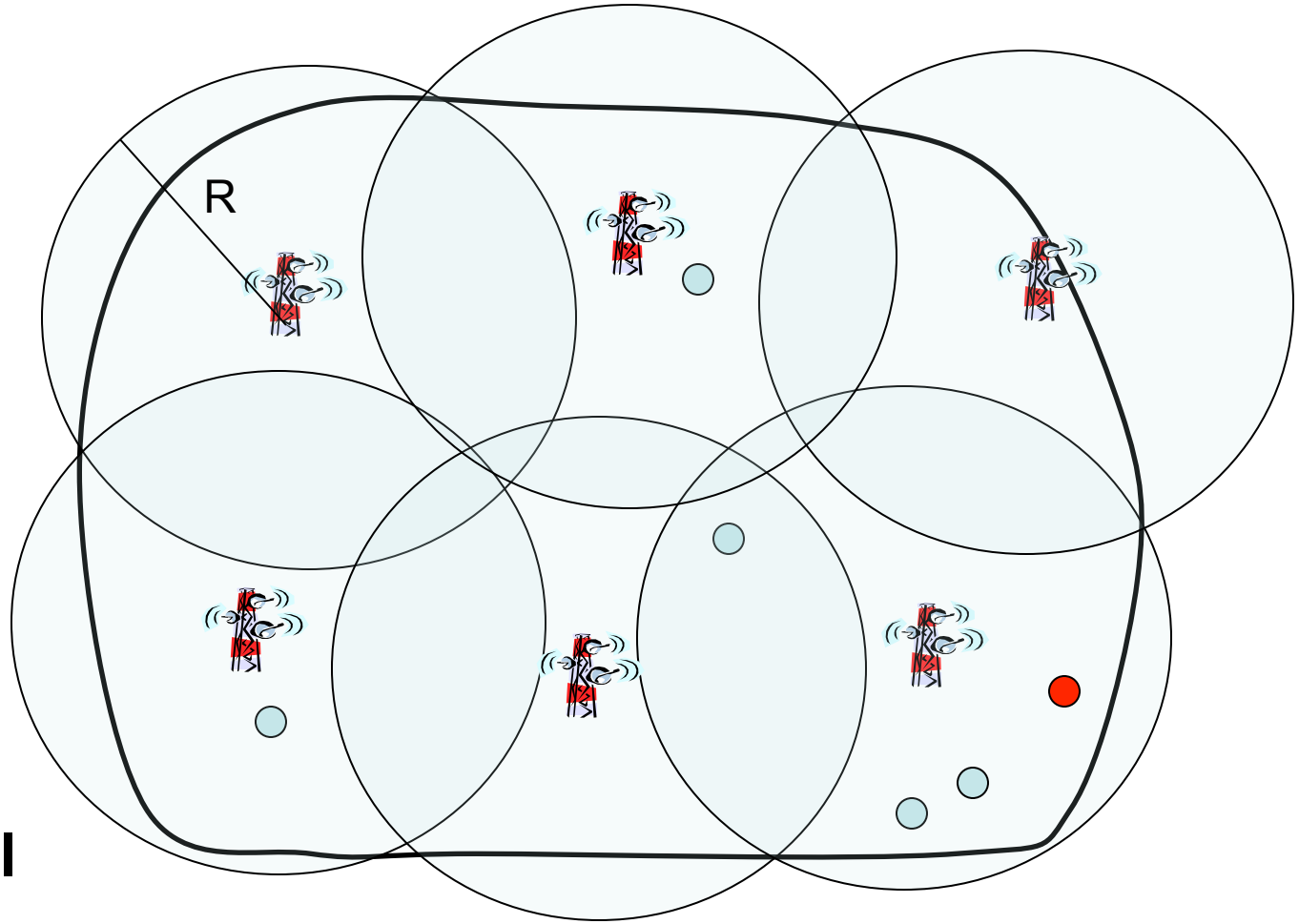
Area Coverage: Cells

Ex.:

N RUs — **N = 12**

L links — **L = 6**

Z cells — **Z = 6**



n RUs/cell — **n = 2**

Splitting resources in separate pools is bad!

Area Coverage: Reuse

Ex.:

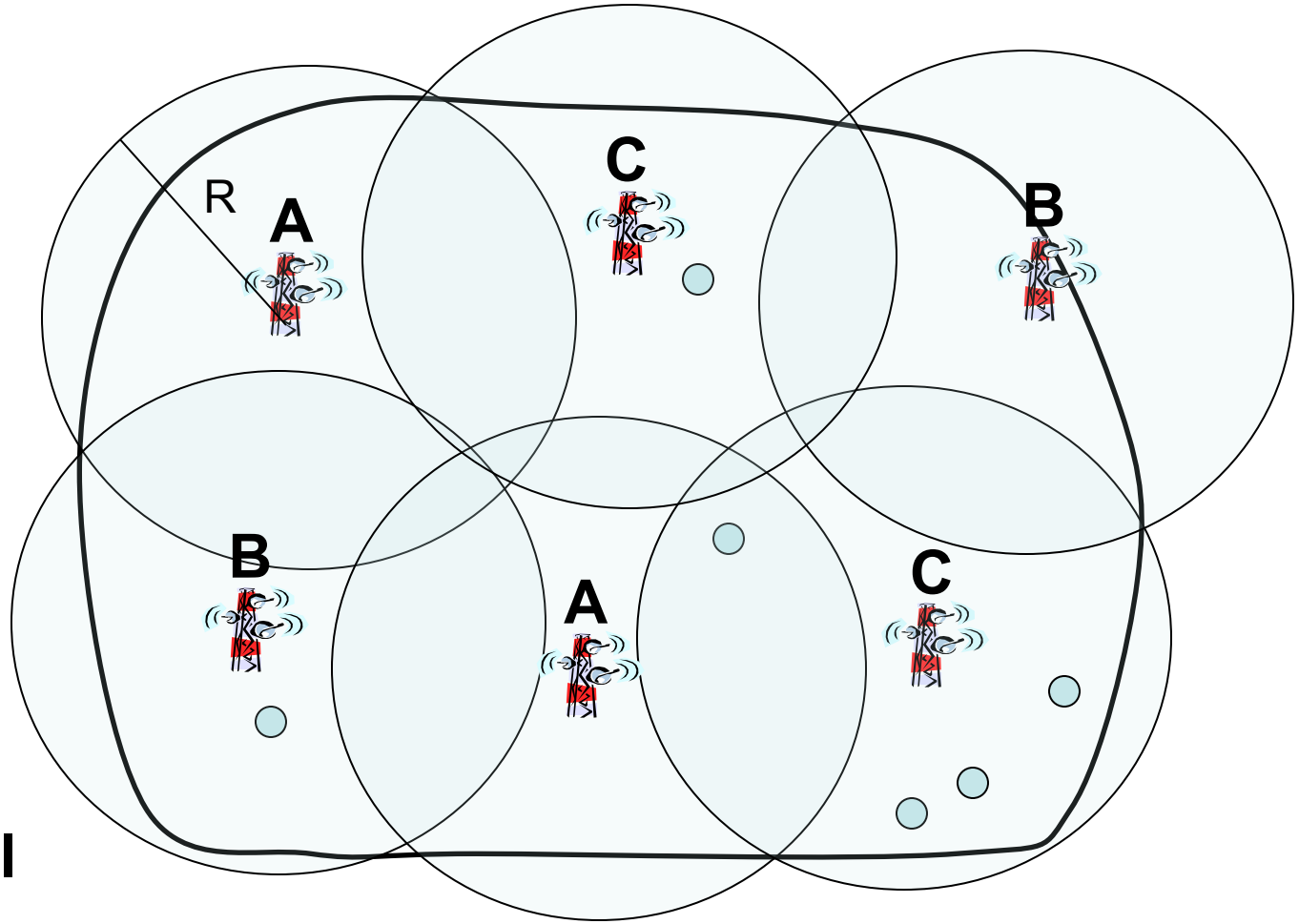
N RUs — N = 12

L links — L = 6

Z cells — Z = 6

K cells / cluster — K = 3

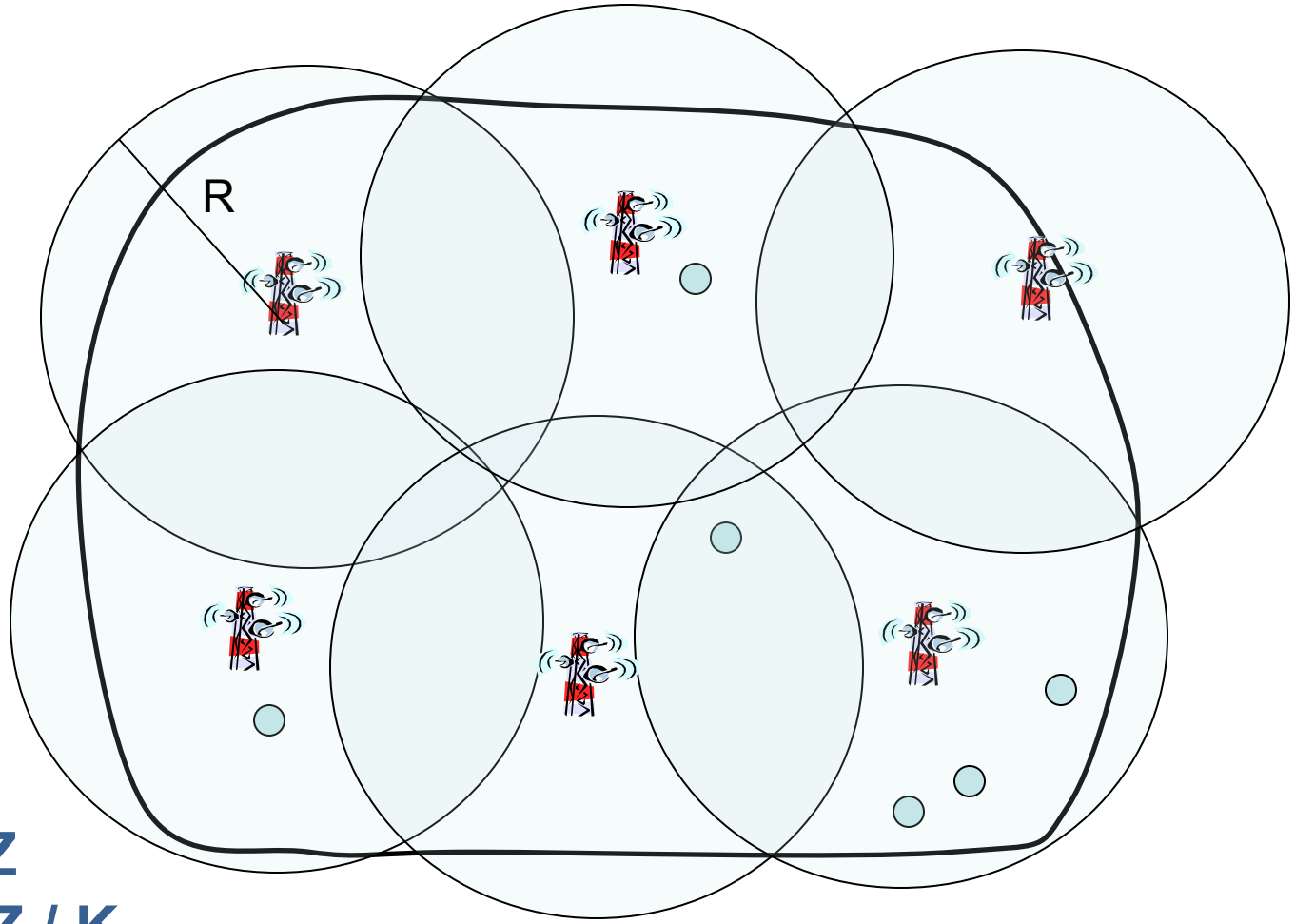
n RUs/cell



$n = N / K$

n = 4

Area Coverage: Reuse



$C_{net} = n*Z$

$= N*Z / K$

Reuse improves Network Capacity!

Radio Resource Utilisation

N	number of RUs available to the system
O	offered traffic within cell (traffic intensity) [Erlang/cell]*
o = O / n	offered traffic within cell per RU [Erlang/cell/RU]*
E	carried traffic within cell [Erlang/cell]*
e = E / n	carried traffic within cell per RU [Erlang/cell/RU]*
e ≤ 1	

In an area coverage network providing voice call services,
e represents Radio Resource Utilisation,
and should be as LARGE as possible from
the network operator pov.

e can be interpreted as the percentage of time a RU in a cell is utilised

* Erlang = traffic unity; it is the amount of traffic that can be carried by one RU

Radio Resource Utilisation

The simplest way to measure the Radio Resource Utilisation is through the B Erlang formula (assuming a blocking system):

$$E = O (1 – Pb)$$

n number of RUs within cell
Pb blocking probability

$$Pb = B [O ; n] = [O^n / n!] / [\Sigma^n_{j=0} O^j / j!]$$

if:
the number of users is infinite
arrivals follow a Poisson Time Process (rate λ)
service times follow an i.i.d. negative exponential (rate μ)

O offered traffic within cell $O = \lambda / \mu$

Radio Resource Utilisation

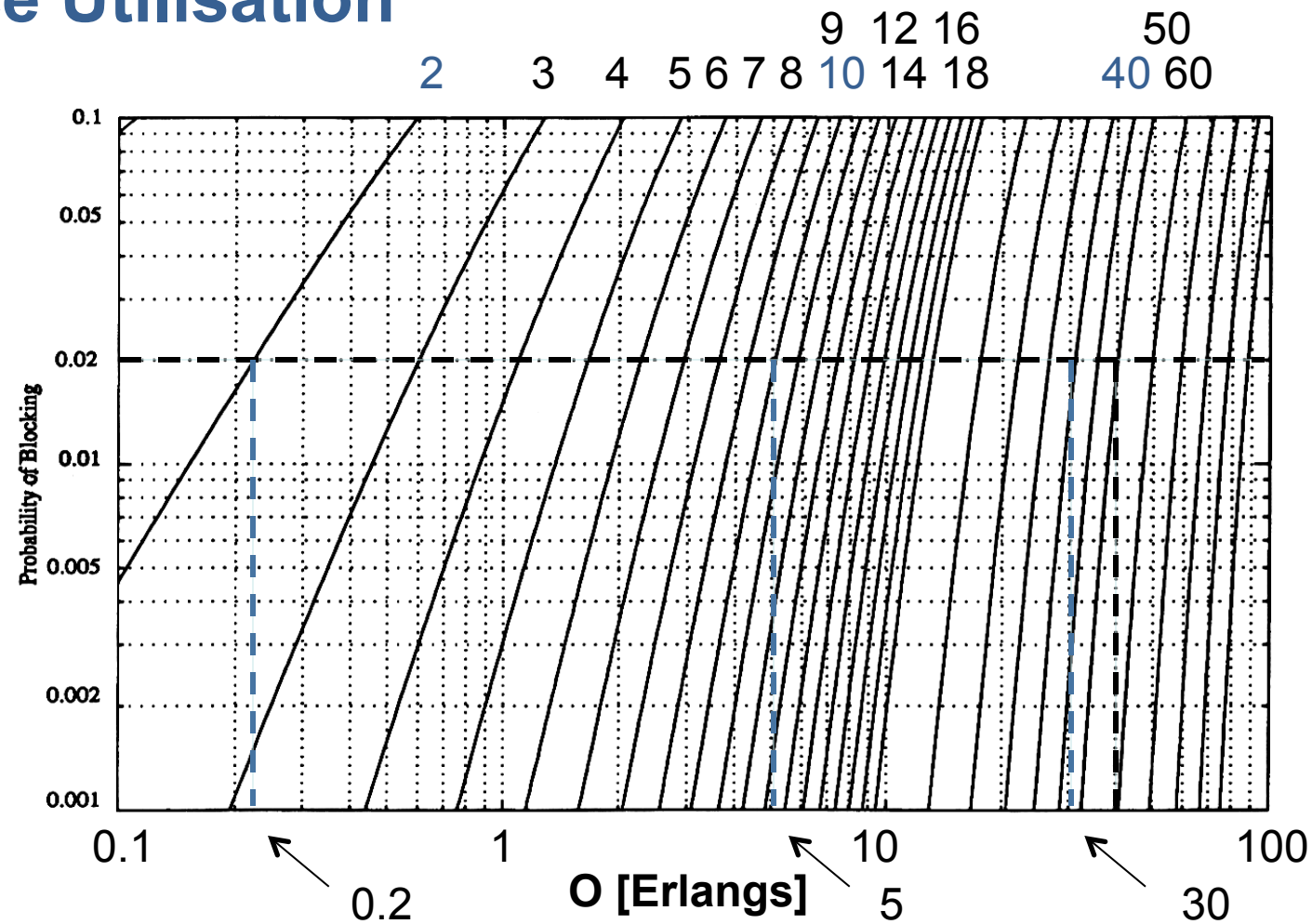
Pb = B [O ; n]

Example:

Pb = 0.02

E = O * 0.98 →
e = o * 0.98 where
o = O / n

n = 2 → e = 0.1
n = 10 → e = 0.5
n = 40 → e = 0.75



Radio Resource Utilisation increases if RUs are assigned to cells in large numbers. Dynamic is better than Fixed RR Assignment schemes.

Splitting Resources in Pools is Always Bad!

Cell Size Dimensioning Based on Capacity

How to set R ?

Pb = 0.02

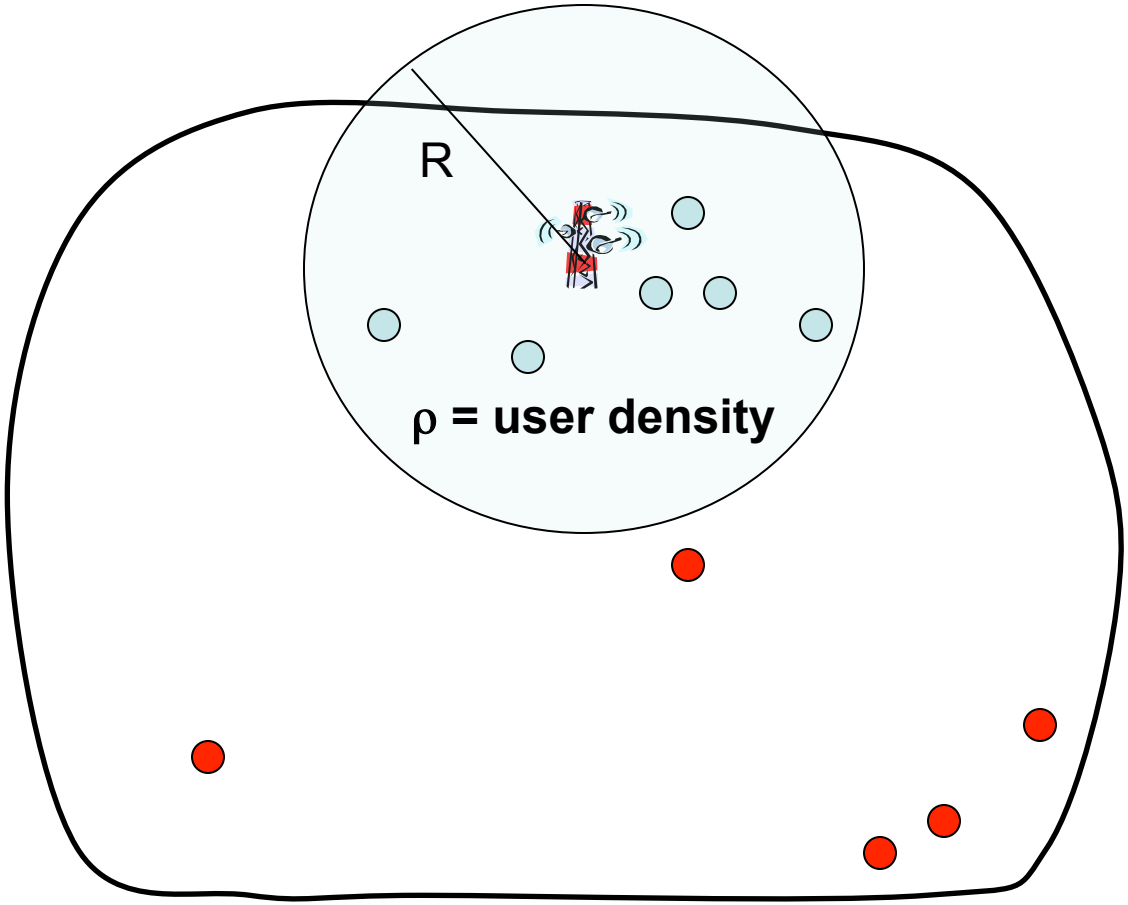
n is fixed

→ O is derived;

O = A * ρ

ρ is fixed

A = π * R²



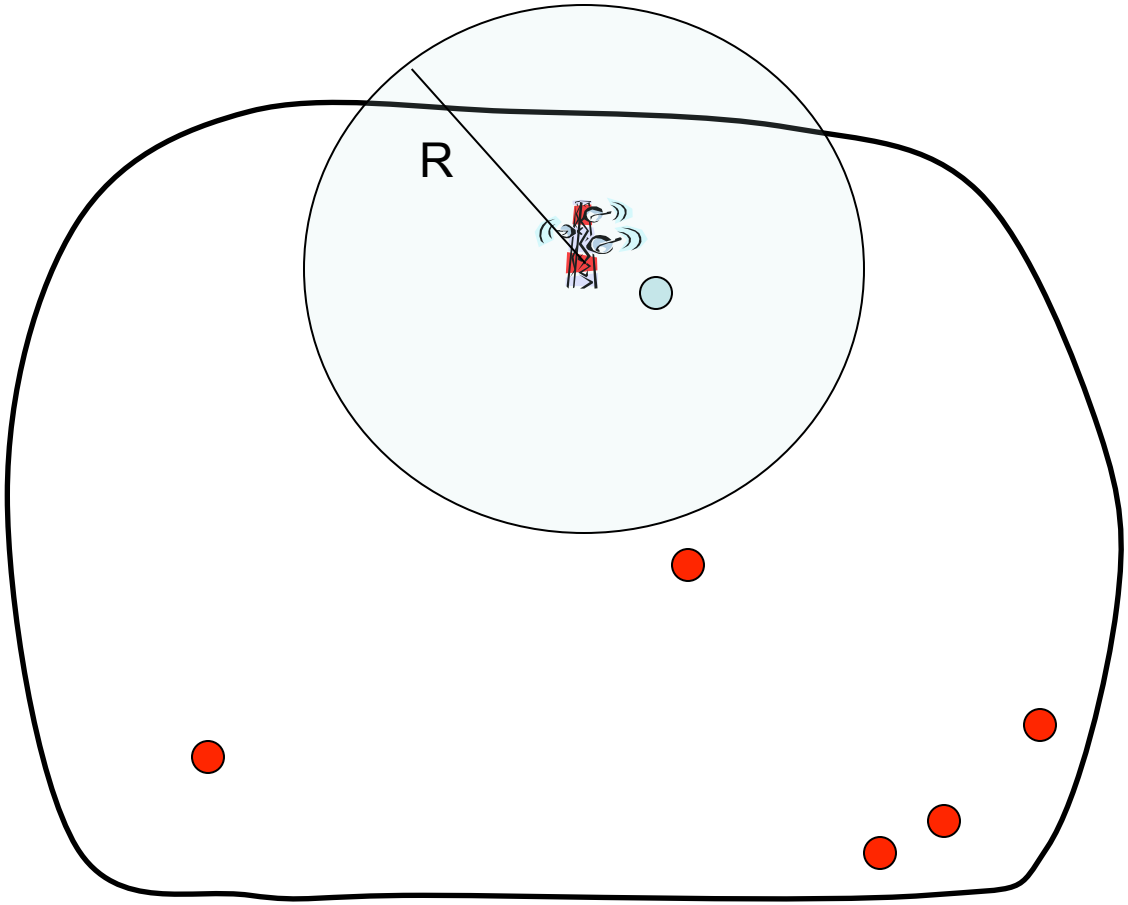
→ R = R_{cap_max} is derived

Cell Size Dimensioning Based on Capacity: Exercise

Assume:

Pb = 0.02
n = 60
ρ = 50 Erl/sqKm

R = ?



….

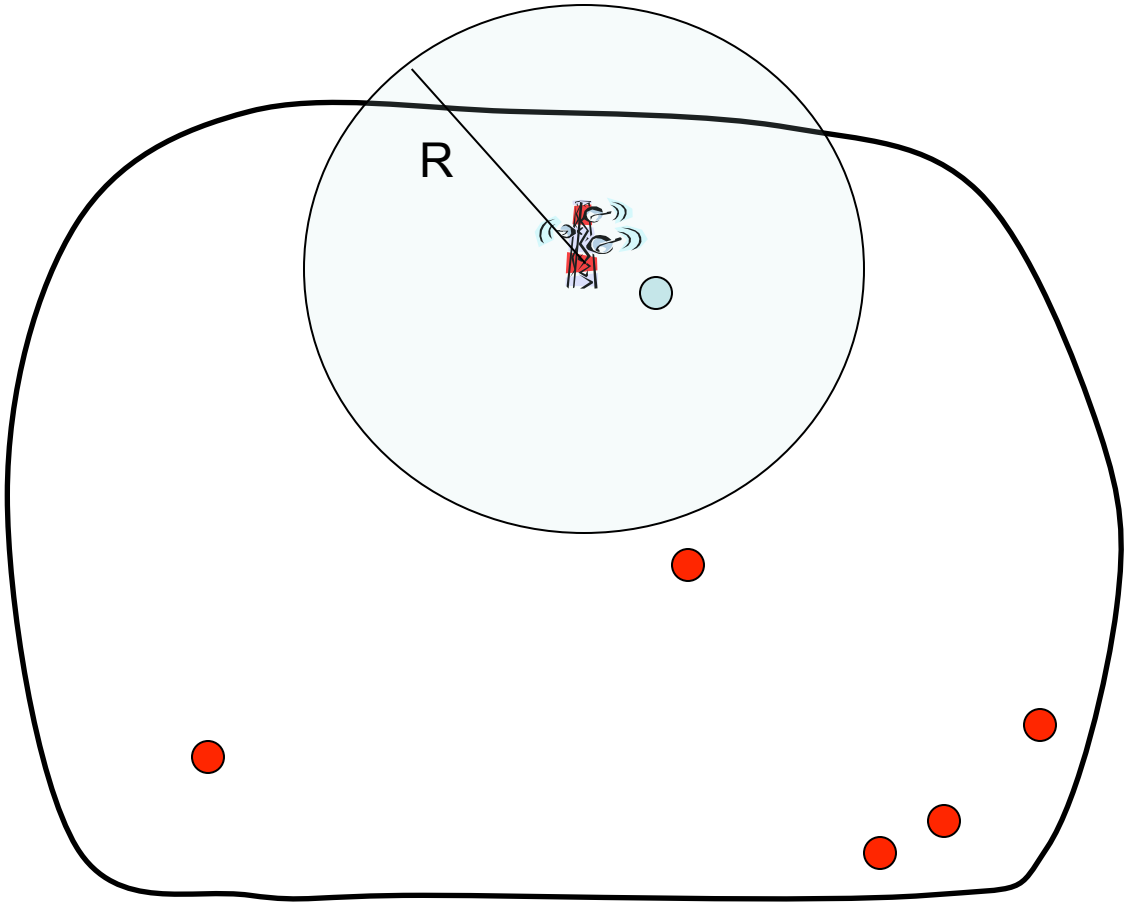
R = 0,500 Km

Cell Size Dimensioning Based on Capacity: Exercise

Assume:

Pb = 0.02
n = 10
ρ = 10 Erl/sqKm

R = ?



Pb = 0.02
n = 40
ρ = 40 Erl/sqKm

R = ?

Area Coverage: Procedure

1) Coverage Based Cell Dimensioning ($R < R_{cov_max}$)

2) Identify Number of Cells

3) Capacity Based Cell Dimensioning ($R < R_{cap_max}$)

Area Coverage: Procedure

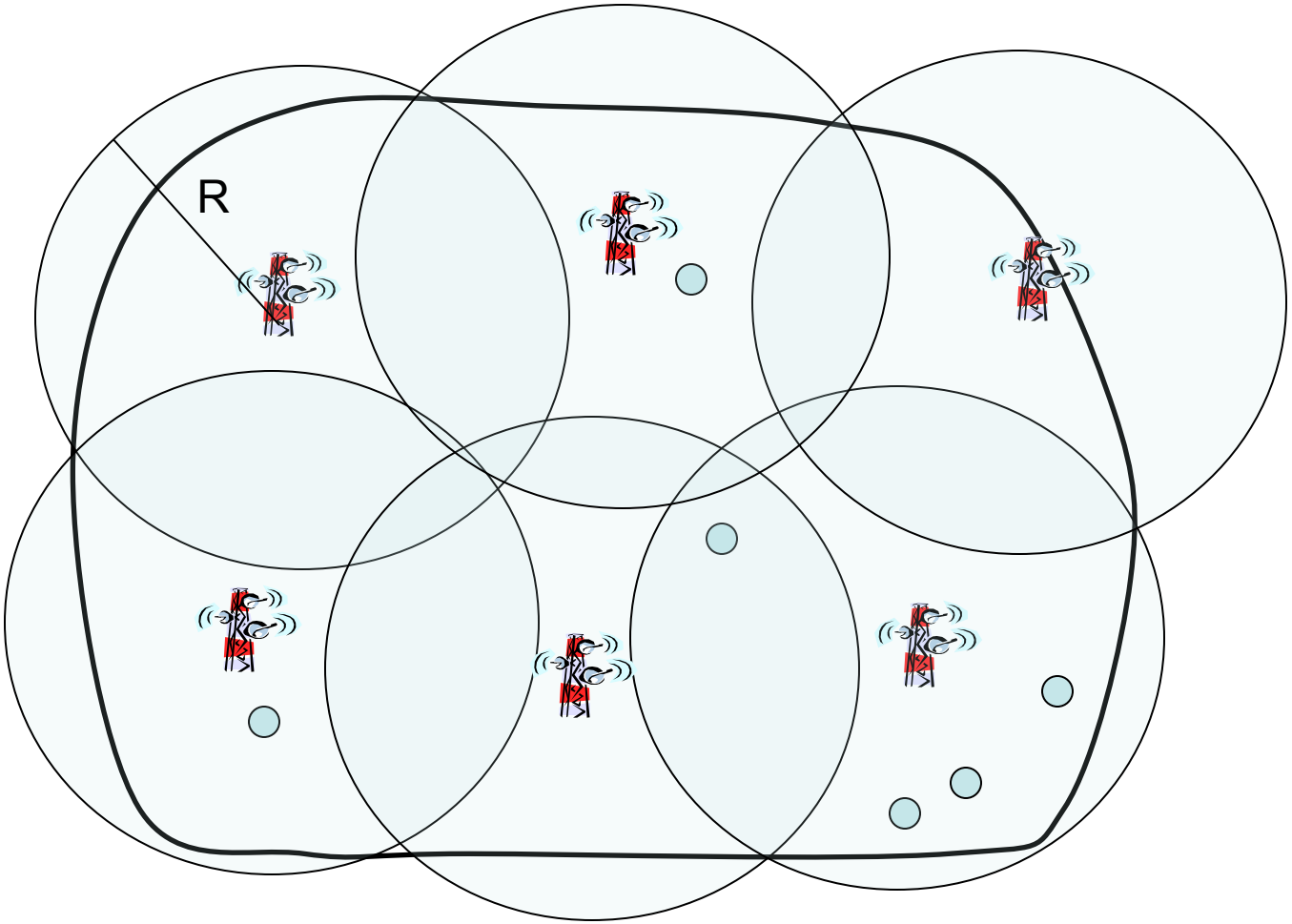
1) Coverage Based Cell Dimensioning ($R < R_{cov_max}$)

2) Identify Number of Cells

3) Capacity Based Cell Dimensioning ($R < R_{cap_max}$)

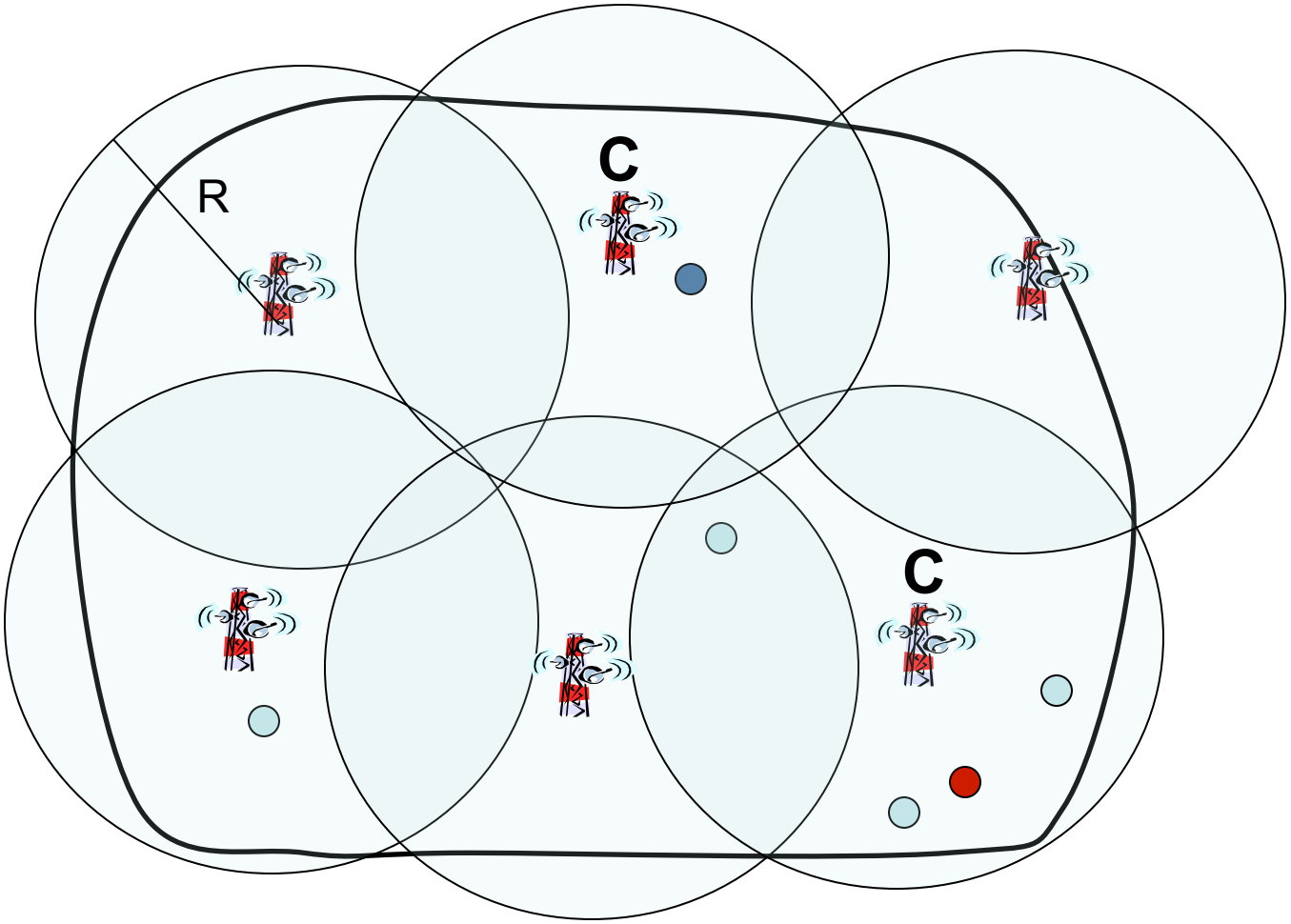
4) $R < \min (R_{cov_max} , R_{cap_max})$ → Network Capacity

Area Coverage: Reuse



Reuse improves Network Capacity …

Area Coverage: Reuse



…at the cost of inter-cell interference!

Area Coverage: Procedure

1) Coverage Based Cell Dimensioning ($R < R_{cov_max}$)

2) Identify Number of Cells

3) Choose Cluster Size

4) Capacity Based Cell Dimensioning ($R < R_{cap_max}$)

5) $R < \min (R_{cov_max} , R_{cap_max})$

2. Reuse

Reuse

N orthogonal RUs

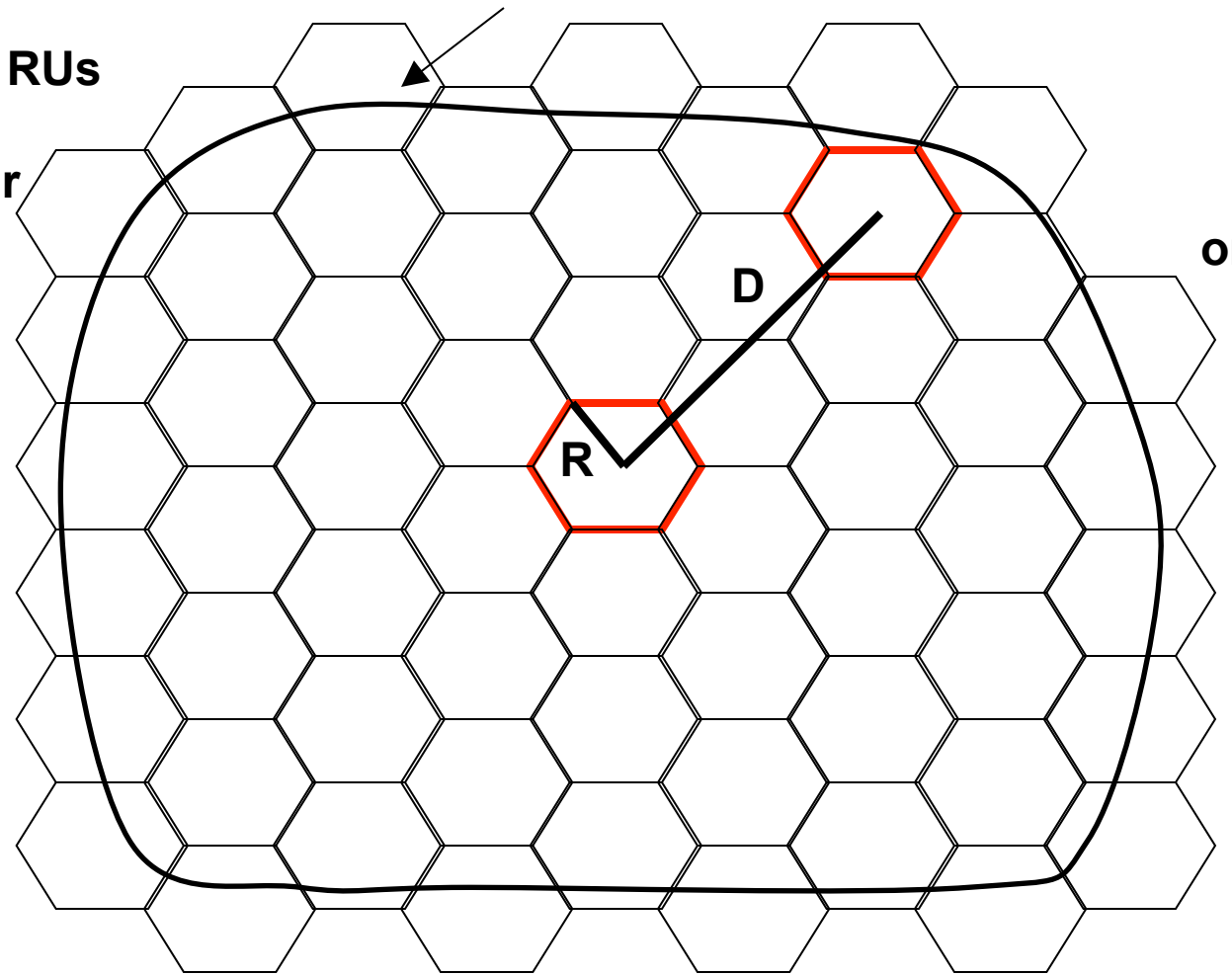
K cells/cluster

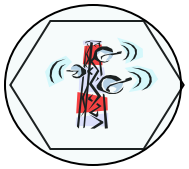
$n = N / K$
RUs/cell

$A = 3^{3/2} R^2 / 2$
cell area

$Q = D / R$
Reuse Factor

Z cells





One base per cell with omnidirectional antenna system

FDD

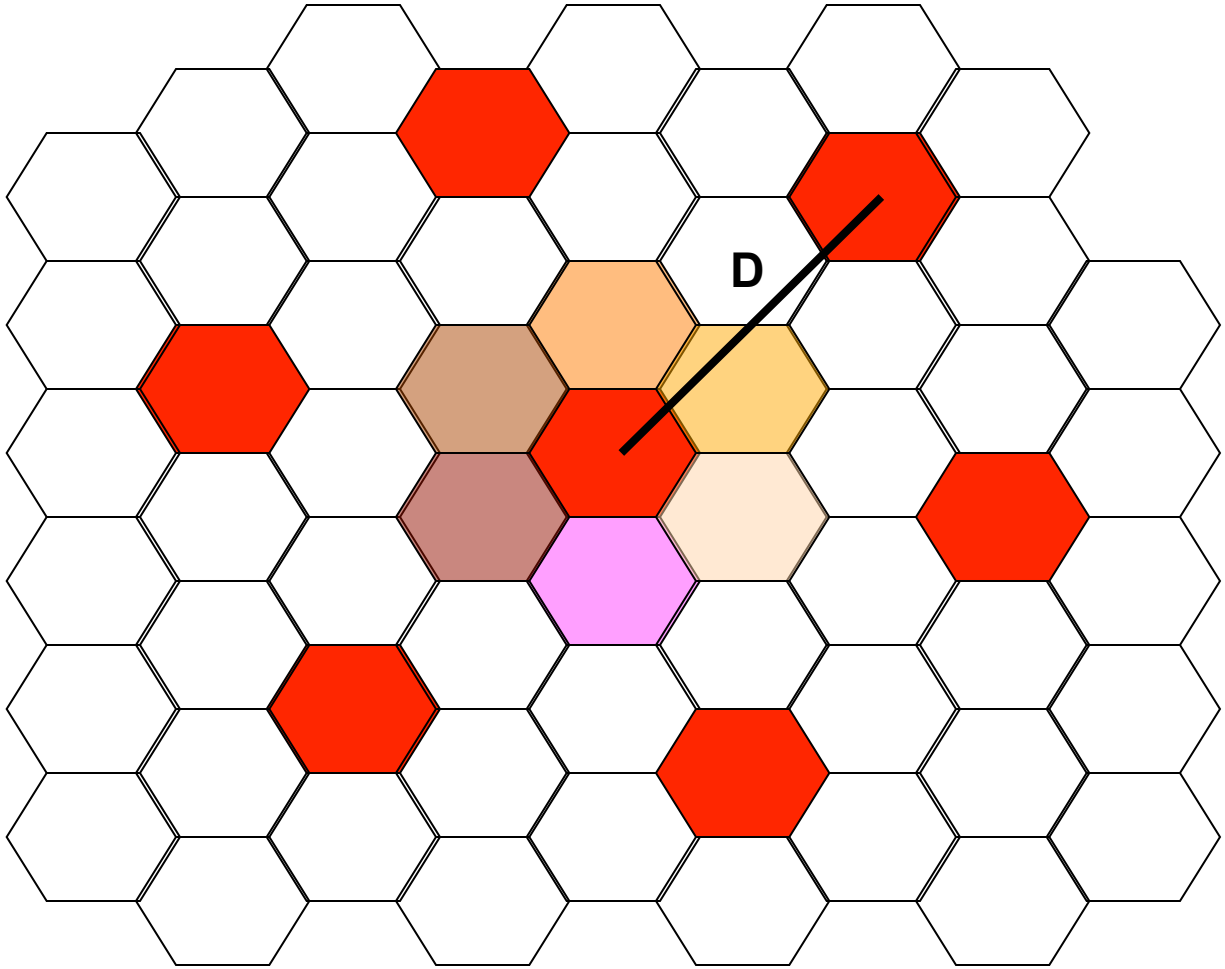
RR Set Capacity: $Z\,n = Z\,N / K$

Signal-to-Interference Ratio $C / I = ?$

Reuse

K = 7

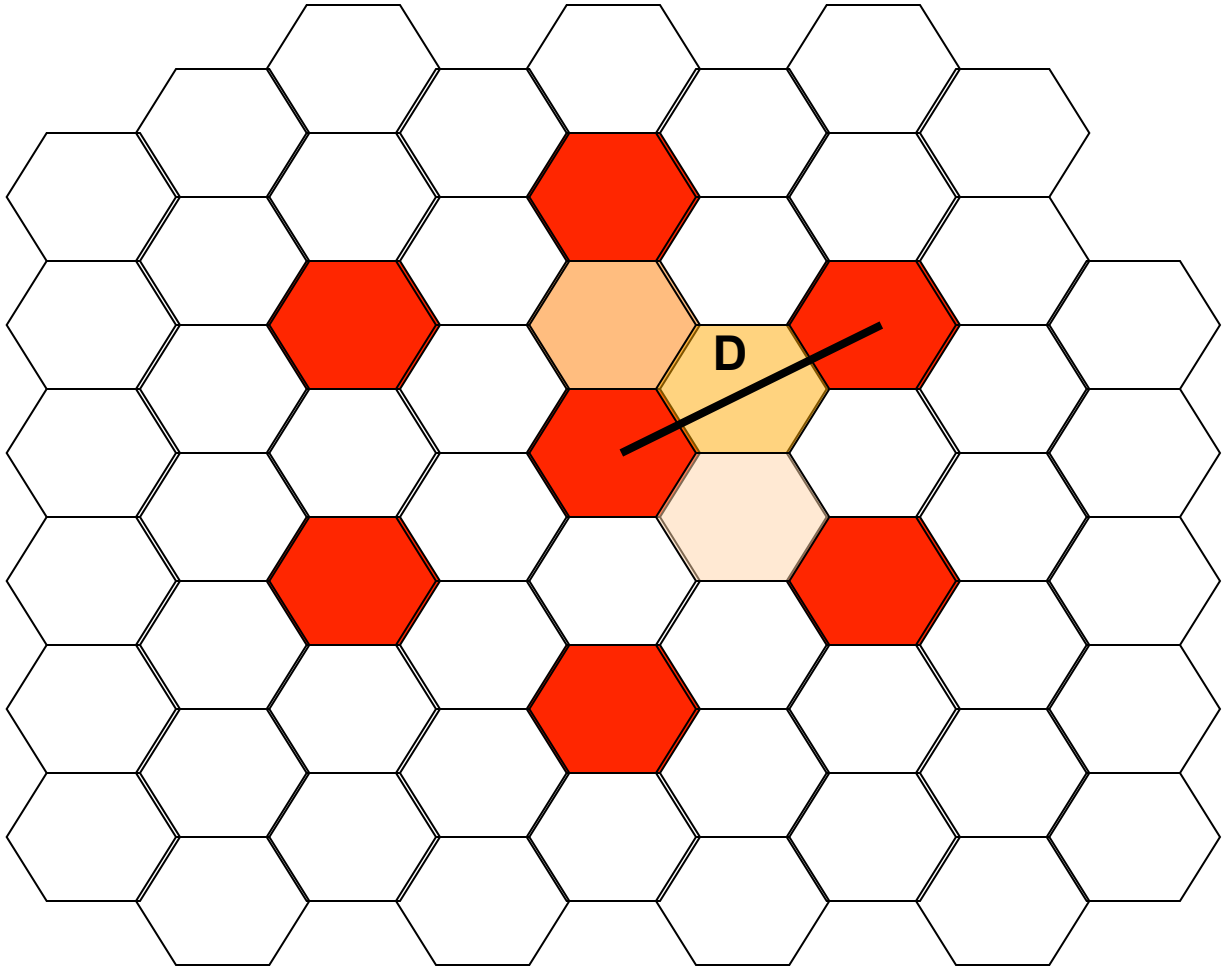
Six first – tier interferers



Reuse

K = 4

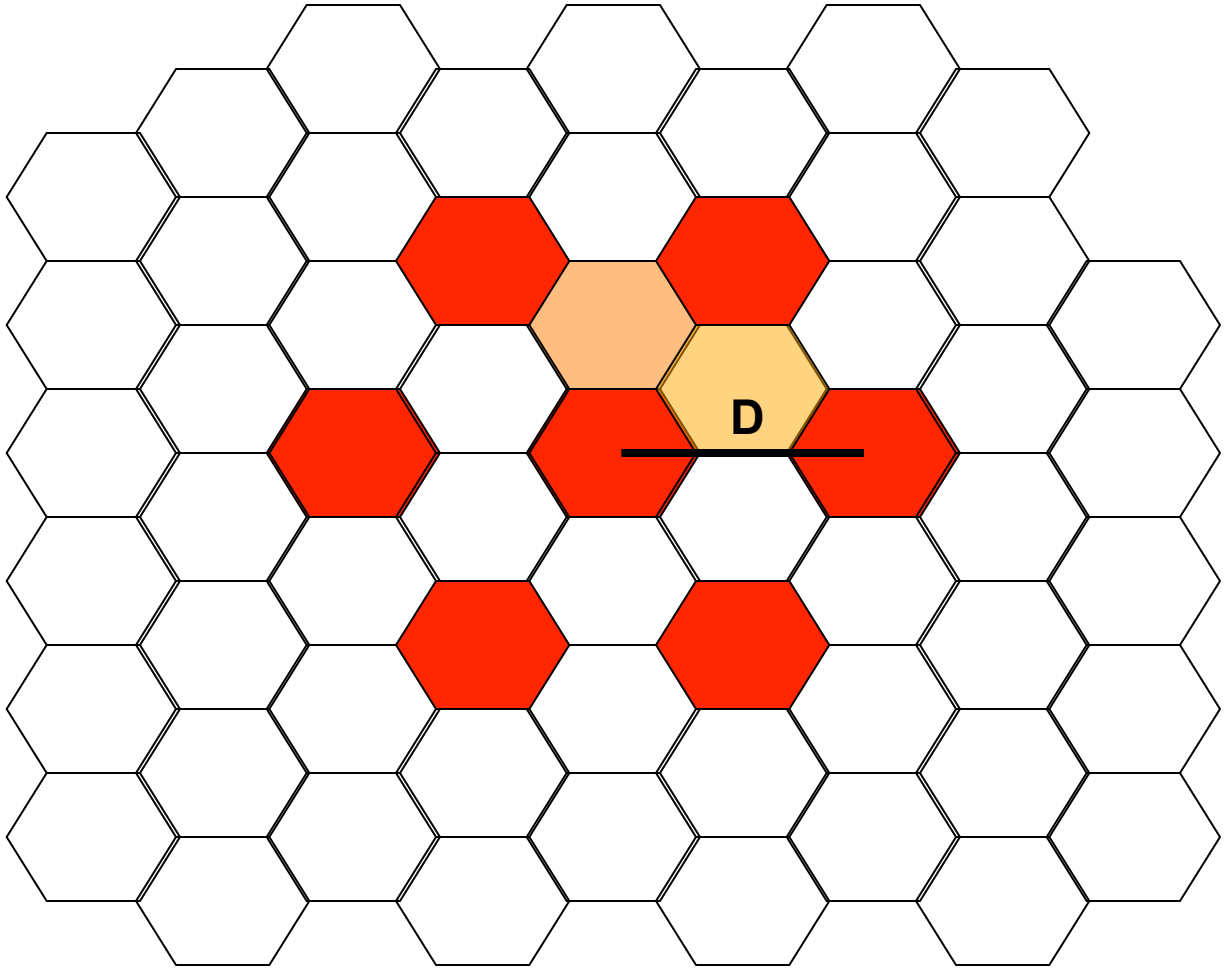
Six first – tier interferers



Reuse

K = 3

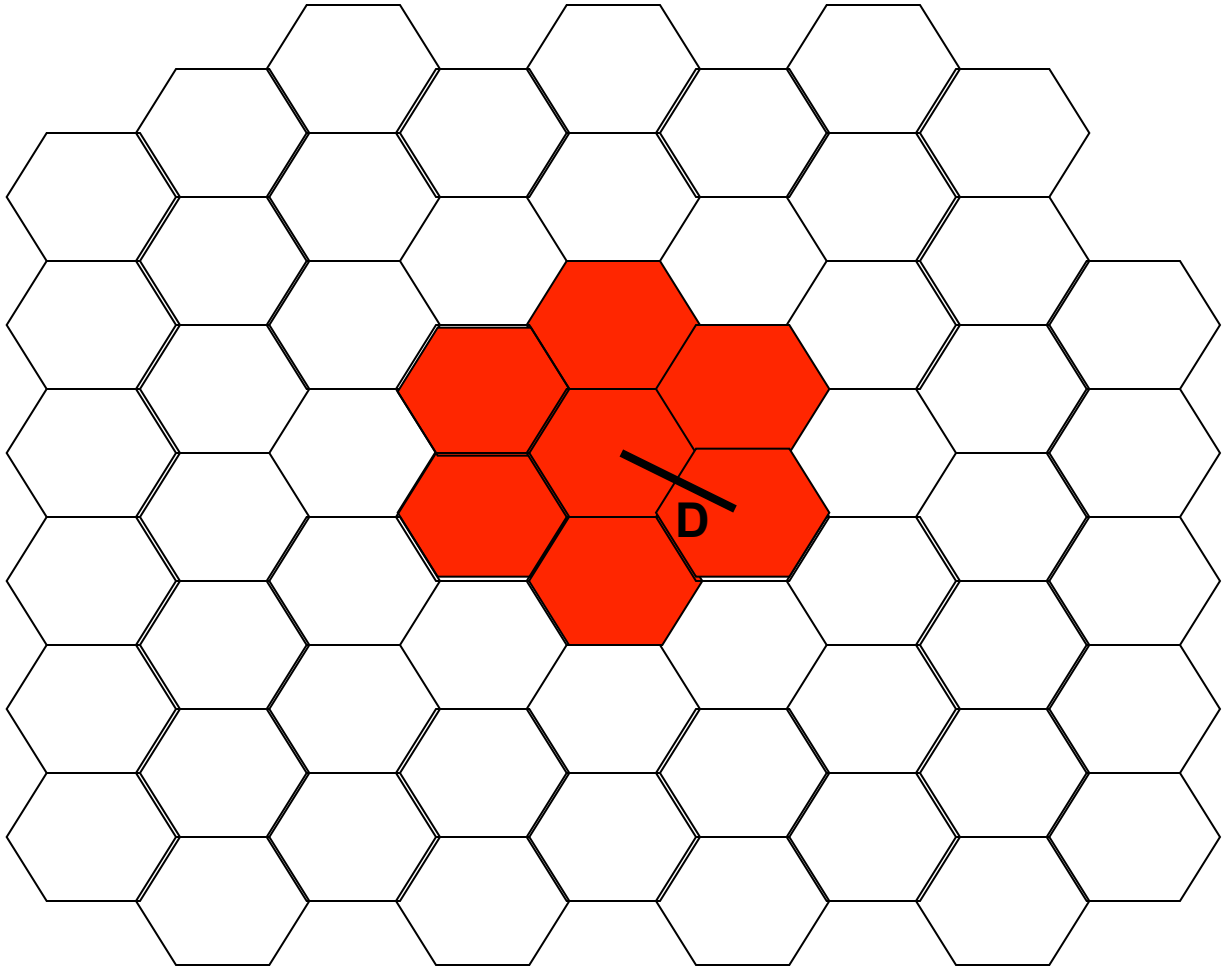
Six first – tier interferers



Reuse

K = 1

Six first – tier interferers



Reuse

$K = i^2 + j^2 + i\,j$ $\qquad$ $i = 0,1,2,3,4,\ldots \quad j = 0,1,2,3,4,\ldots$

i	j	K	
1	0	1	(full reuse)
1	1	3	
2	0	4	
2	1	7	
3	0	9	
2	2	12	

…

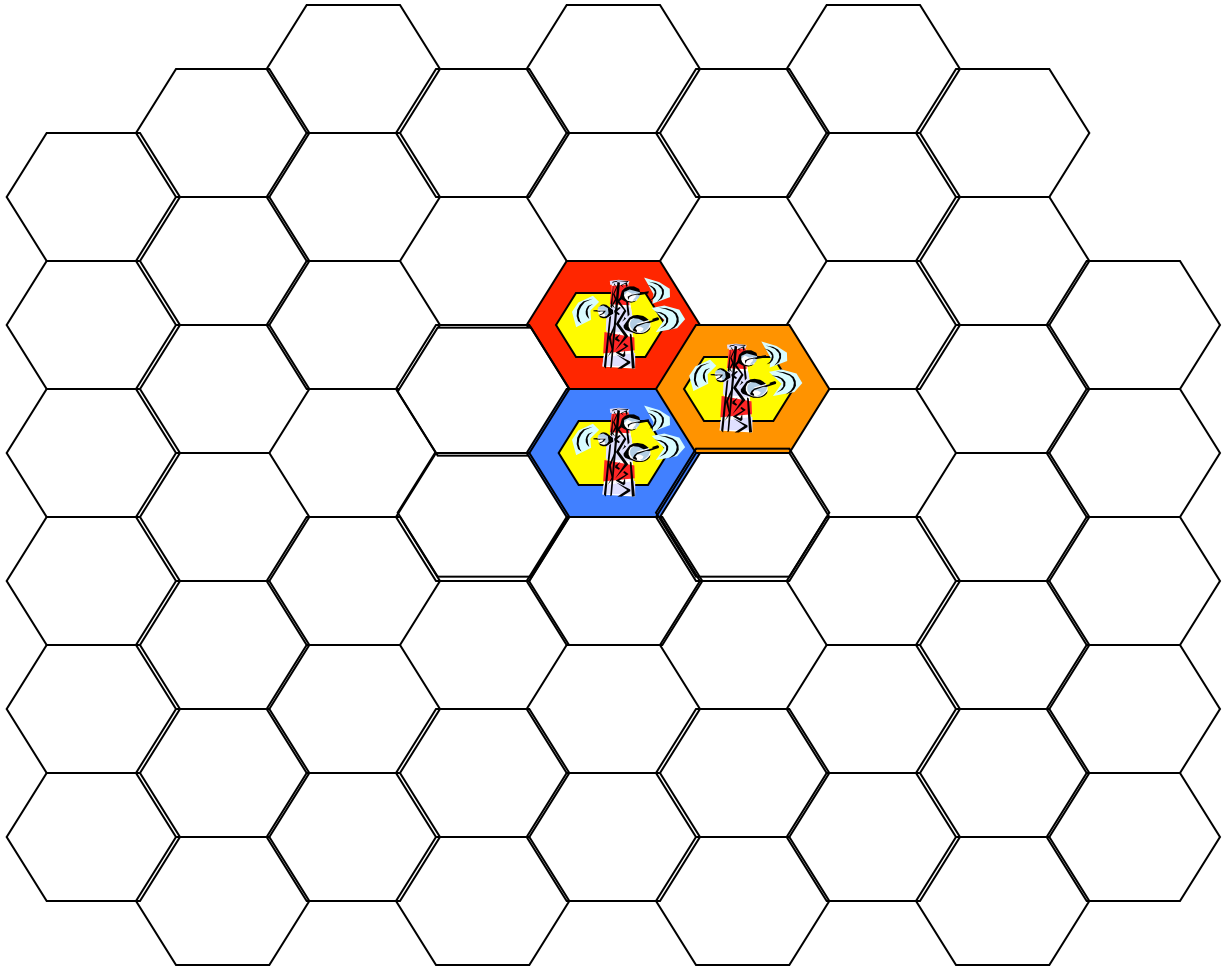
$D = R\,(3\,K)^{1/2}$ $\qquad$ $n = N / K$

$Q = (3\,K)^{1/2}$

Increasing K, D increases, but n decreases
(less interference, at the expense of a smaller capacity)

Reuse: Fractional Reuse

K = 1 / 3



Reuse

$C / I = Q^{\beta} / Nint = (3\ K)^{\beta/2} / Nint$

Nint = 6 (first tier) with omnidirectional antennas at the base stations
Nint = 2 (first tier) with three-sectorial antennas at the base stations

Does this formula holds in all conditions?

Reuse: How to Maximize Capacity?

N orthogonal RUs

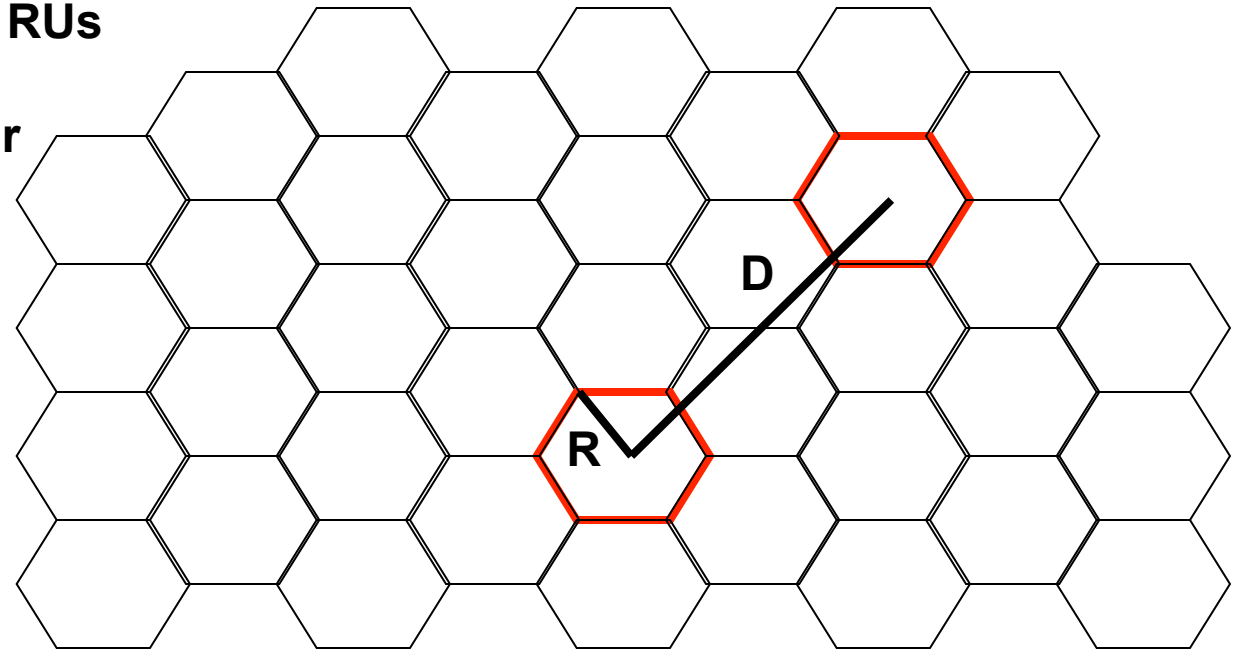
K cells/cluster

$n = N / K$
RUs/cell

$A = 3^{3/2} R^2 / 2$
cell area

$Q = D / R$
Reuse Factor

Z cells



Network Capacity: $Z\,n = Z\,N / K$ (proportional to Z)

$C / I = Q^{\beta} / Nint = (3\,K)^{\beta/2} / Nint$ (independent on Z)

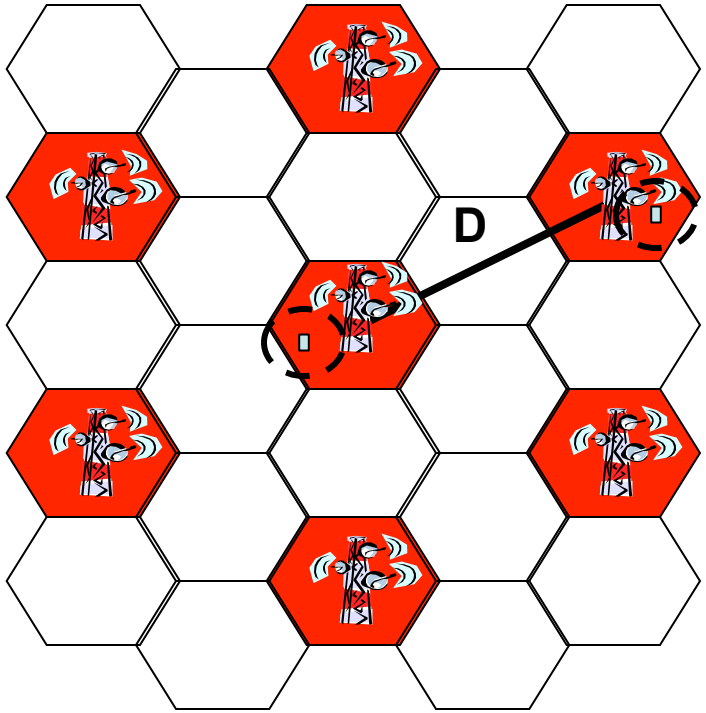
Make cells as small as possible (larger Z)

Carrier-to-Interference Ratio scales!

Reuse: Deterministic Conditions – Omni Antennas

$C = k\, d_u -^{\beta}$
$I = Sum_j\, k\, d_j -^{\beta}$

FDD



$C / I = k\, d_u -^{\beta} / Sum_j\, k\, d_j -^{\beta} = d_u -^{\beta} / Sum_j\, d_j -^{\beta}$

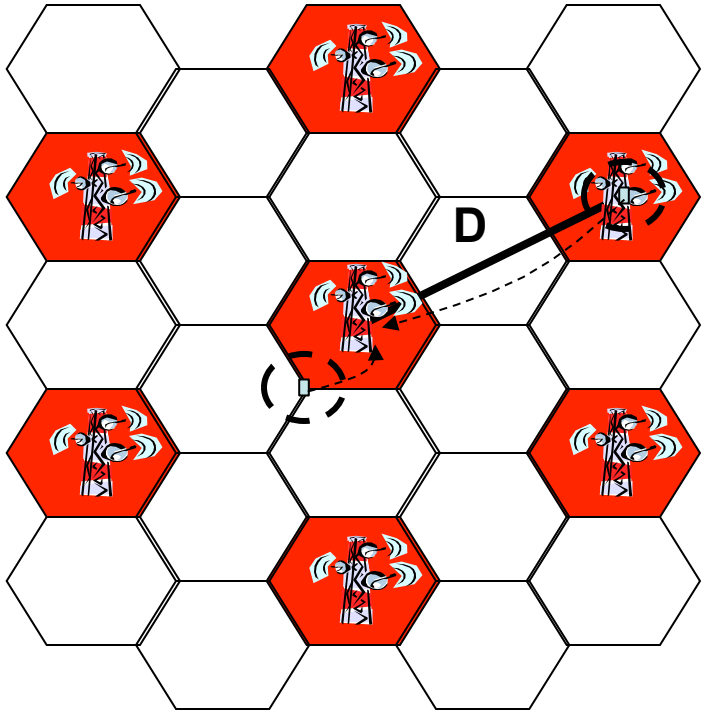
Reuse: Deterministic Conditions – Omni Antennas

$C = k\ R^{-\beta}$ (worst situation)

$I = Nint\ k\ D^{-\beta}$ (average situation)

FDD

uplink



$C / I = Q^{\beta} / Nint = (3\ K)^{\beta/2} / Nint$ (k is not influent!)

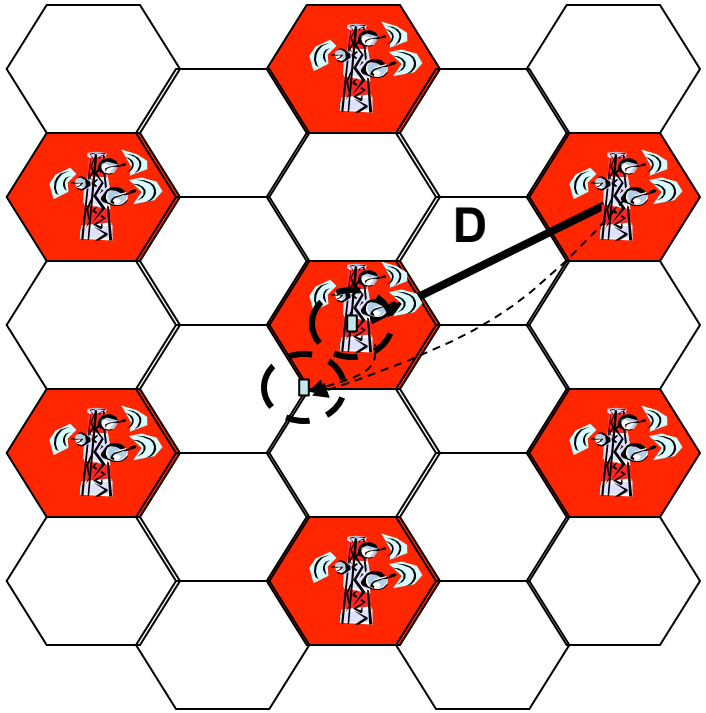
Reuse: Deterministic Conditions – Omni Antennas

$C = k\ R^{-\beta}$ (worst situation)

$I = Nint\ k\ D^{-\beta}$ (average situation)

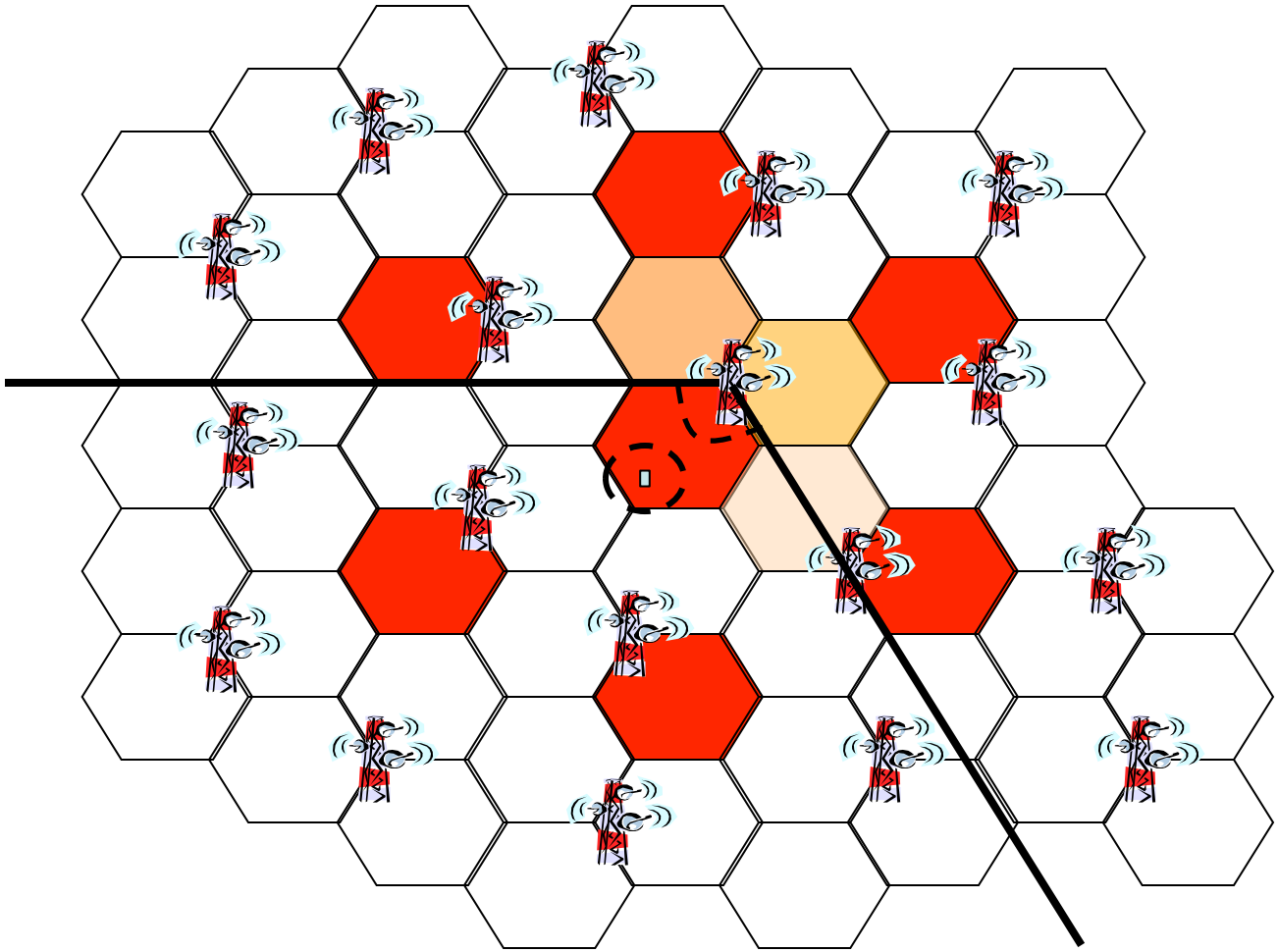
FDD

downlink



$C / I = Q^{\beta} / Nint = (3\ K)^{\beta/2} / Nint$

Reuse: Deterministic Conditions – Tri-Sect Antennas

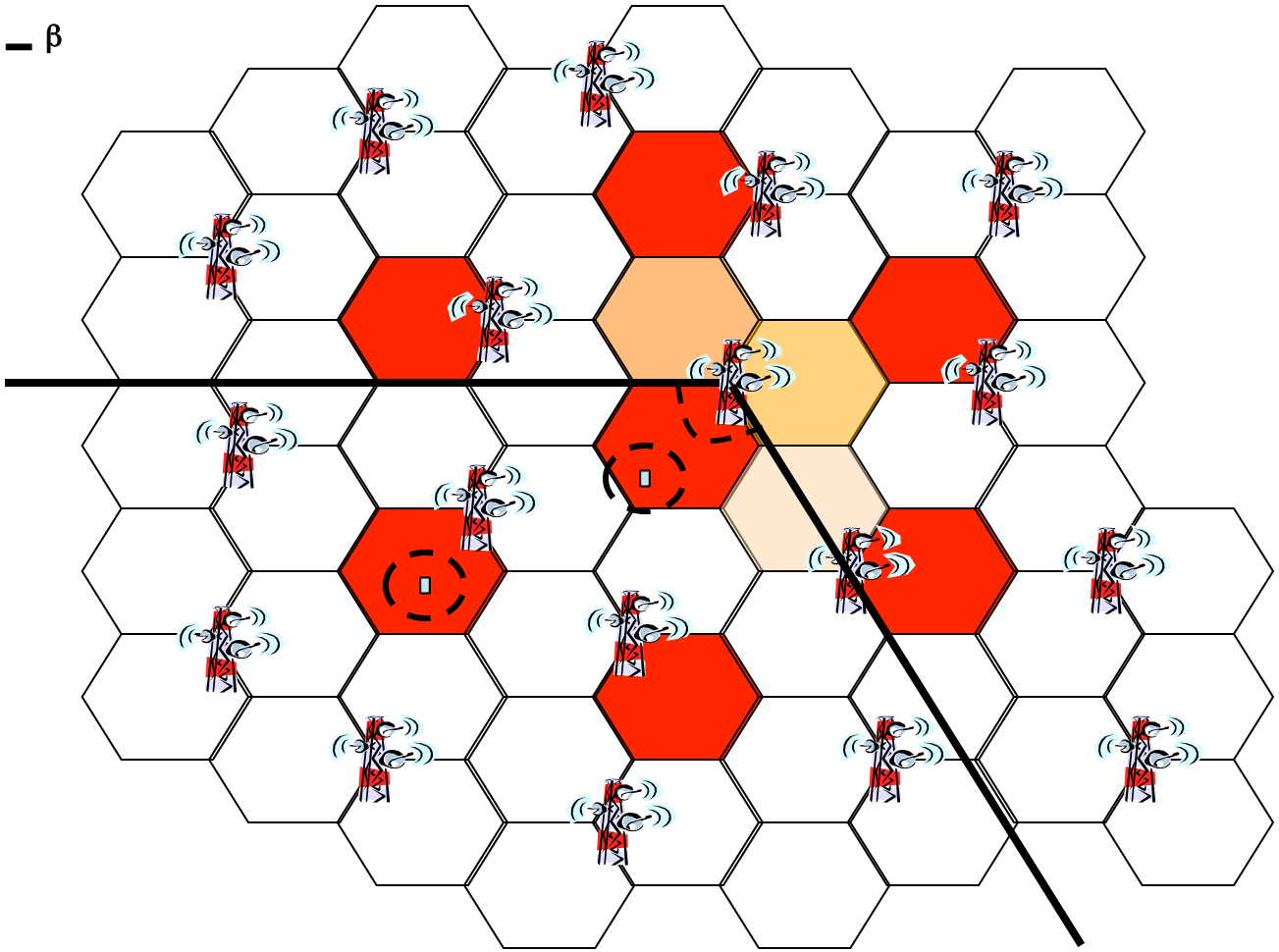


Reuse: Deterministic Conditions – Tri-Sect Antennas

$C = k\, d_u -^{\beta}$

$I = \mathrm{Sum}_j\, k\, d_j -^{\beta}$

FDD



C / I = ?

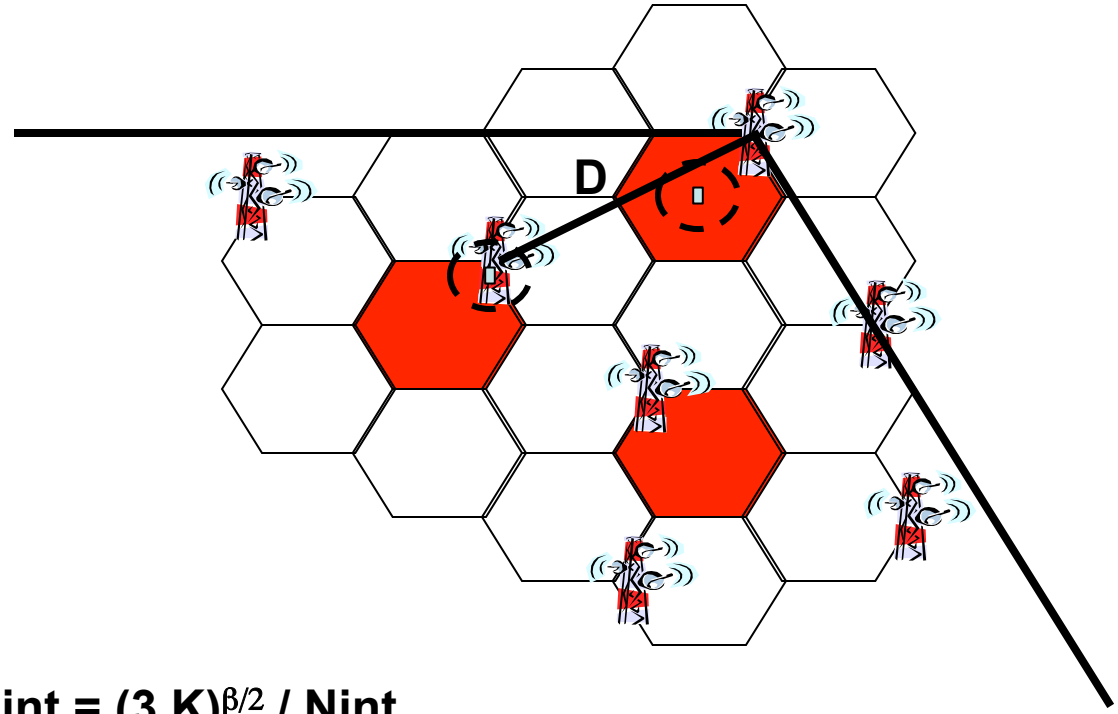
Reuse: Deterministic Conditions – Tri-Sect Antennas

$C = k\ R^{-\beta}$ (average situation)

$I = N_{int}\ k\ D^{-\beta}$ (worst situation)

FDD

uplink



$C / I = Q^{\beta} / N_{int} = (3\ K)^{\beta/2} / N_{int}$

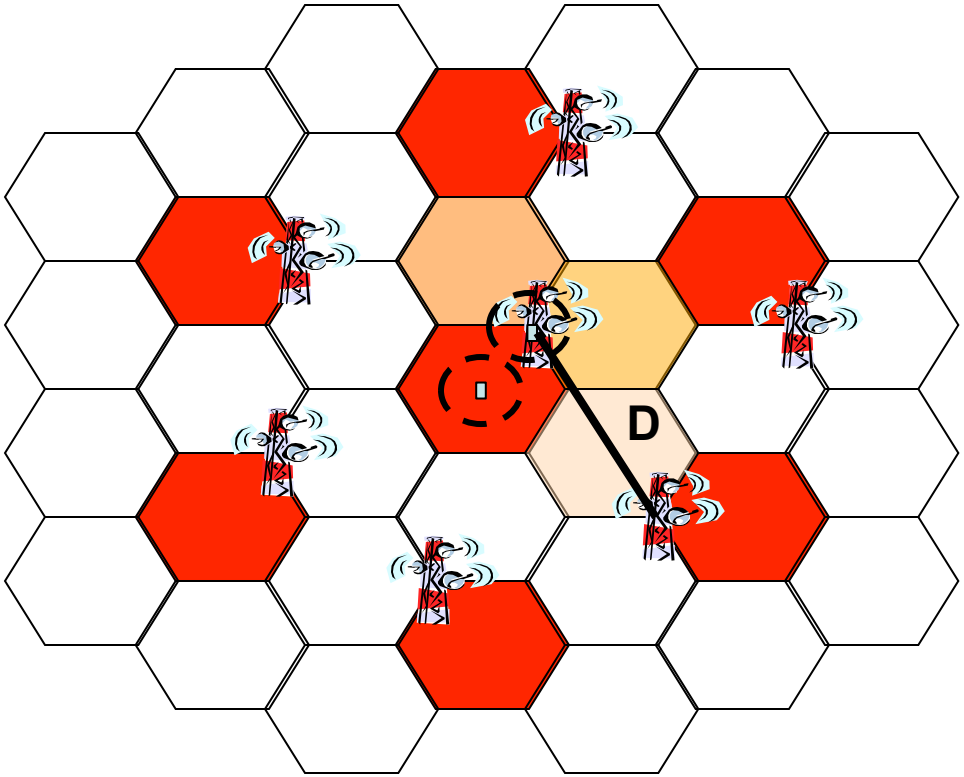
Reuse: Deterministic Conditions – Tri-Sect Antennas

$C = k\ R^{-\beta}$

$I = Nint\ k\ D^{-\beta}$???

FDD

downlink?



$C / I = Q^{\beta} / Nint = (3\ K)^{\beta/2} / Nint$ Nint = ???

There is no general rule!

Reuse: Exercise

Compute the C / I under deterministic conditions with propagation exponent equal to 4, for cluster size equal to 7, 4, 3 with omnidirectional and tri-sectorial antennas. Repeat computation for propagation exponent equal to 3.

Reuse: Exercise

Compute the C / I under deterministic conditions with propagation exponent equal to 4, for cluster size equal to 7, 4, 3 with omnidirectional and tri-sectorial antennas. Repeat computation for propagation exponent equal to 3.

$C / I = (3 K)^{\beta/2} / Nint$

	Omnidirectional antennas	Tri-sectorial antennas
$\beta = 4$		
K = 7	18.7 dB	23.4 dB
K = 4	13,8 dB	18.6 dB
K = 3	11.3 dB	16.1 dB
$\beta = 3$		
K = 7	12.1 dB	16.9 dB
K = 4	8.4 dB	13.2 dB
K = 3	6.5 dB	11.3 dB

Reuse: Exercise

Compute the C / I under deterministic conditions with propagation exponent equal to 4, for cluster size equal to 7, 4, 3 with omnidirectional and tri-sectorial antennas. Repeat computation for propagation exponent equal to 3.

$C / I = (3 K)^{\beta/2} / Nint$ **Assume α = 10 dB**

	Omnidirectional antennas	Tri-sectorial antennas
$\beta = 4$		
K = 7	18.7 dB	23.4 dB
K = 4	13,8 dB	18.6 dB
K = 3	11.3 dB	16.1 dB
$\beta = 3$		
K = 7	12.1 dB	16.9 dB
K = 4	8.4 dB	13.2 dB
K = 3	6.5 dB	11.3 dB

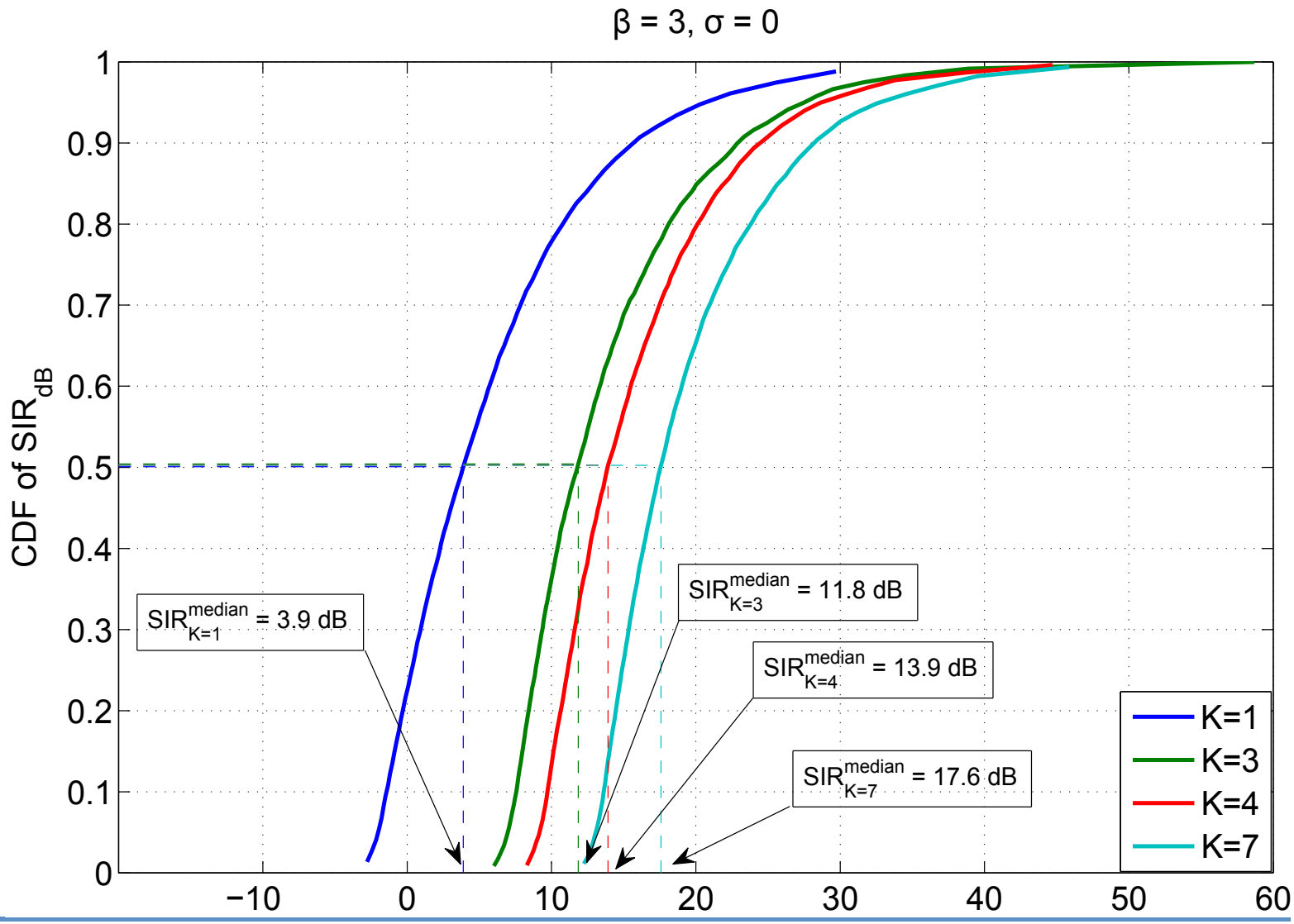
Reuse: Simulations

Compute the (average) C / I under average conditions with propagation exponent equal to 4, for cluster size equal to 7, 4, 3 with tri-sectorial antennas.
Repeat computation for propagation exponent equal to 3.

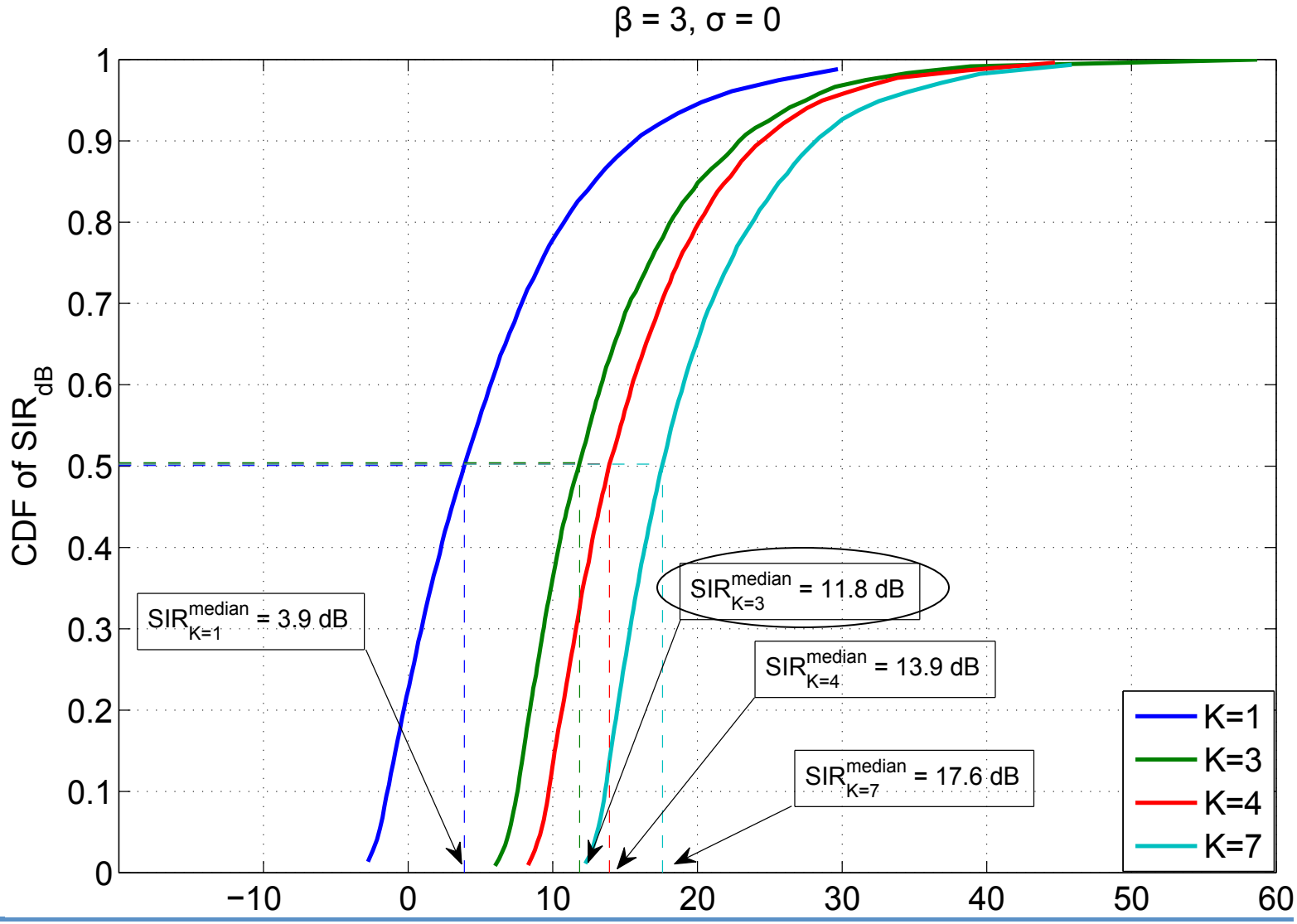
Under average conditions, users are randomly and uniformly distributed in the cells.

Repeat computation including shadowing with standard deviation 6 or 12.

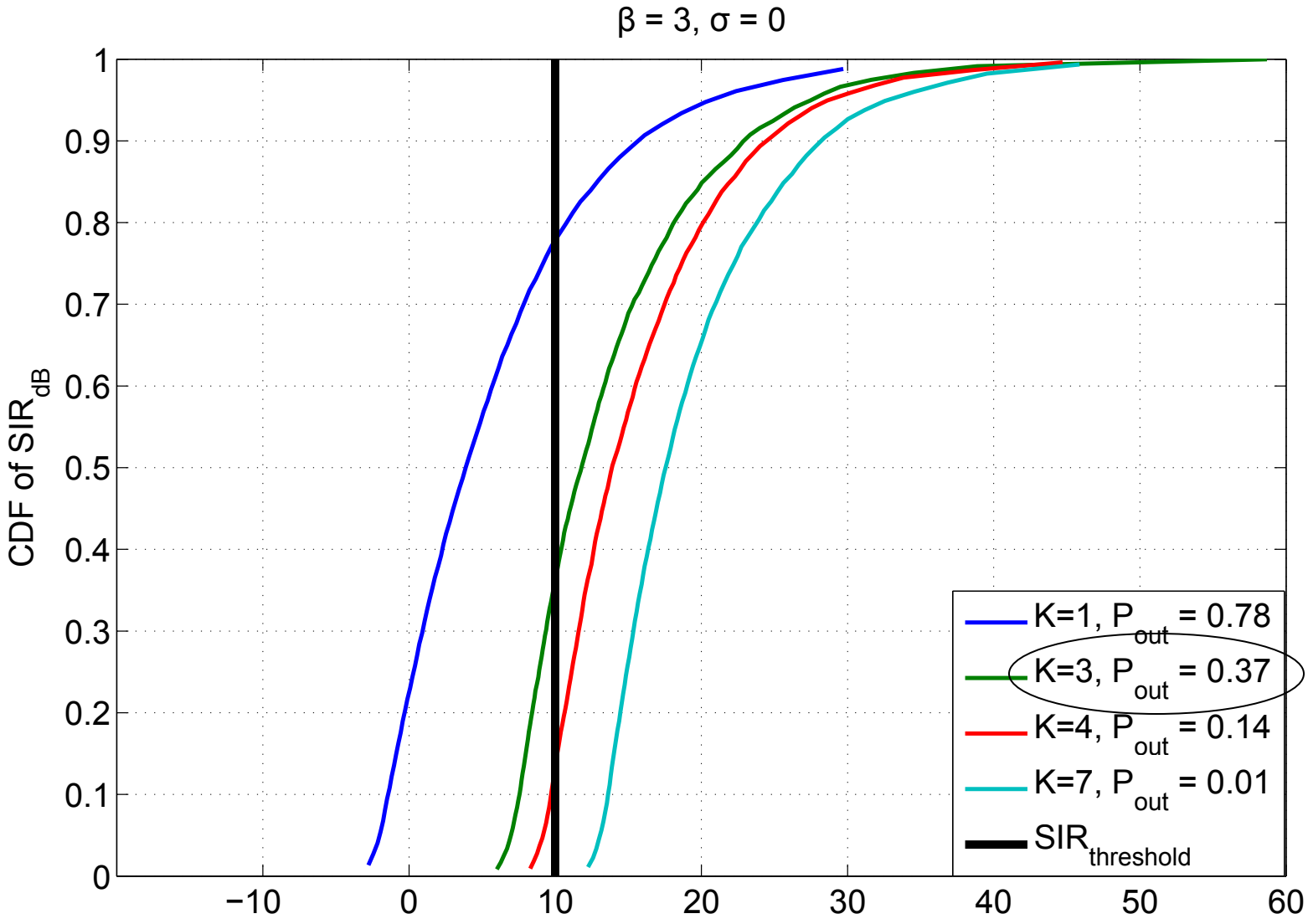
Reuse: Simulations



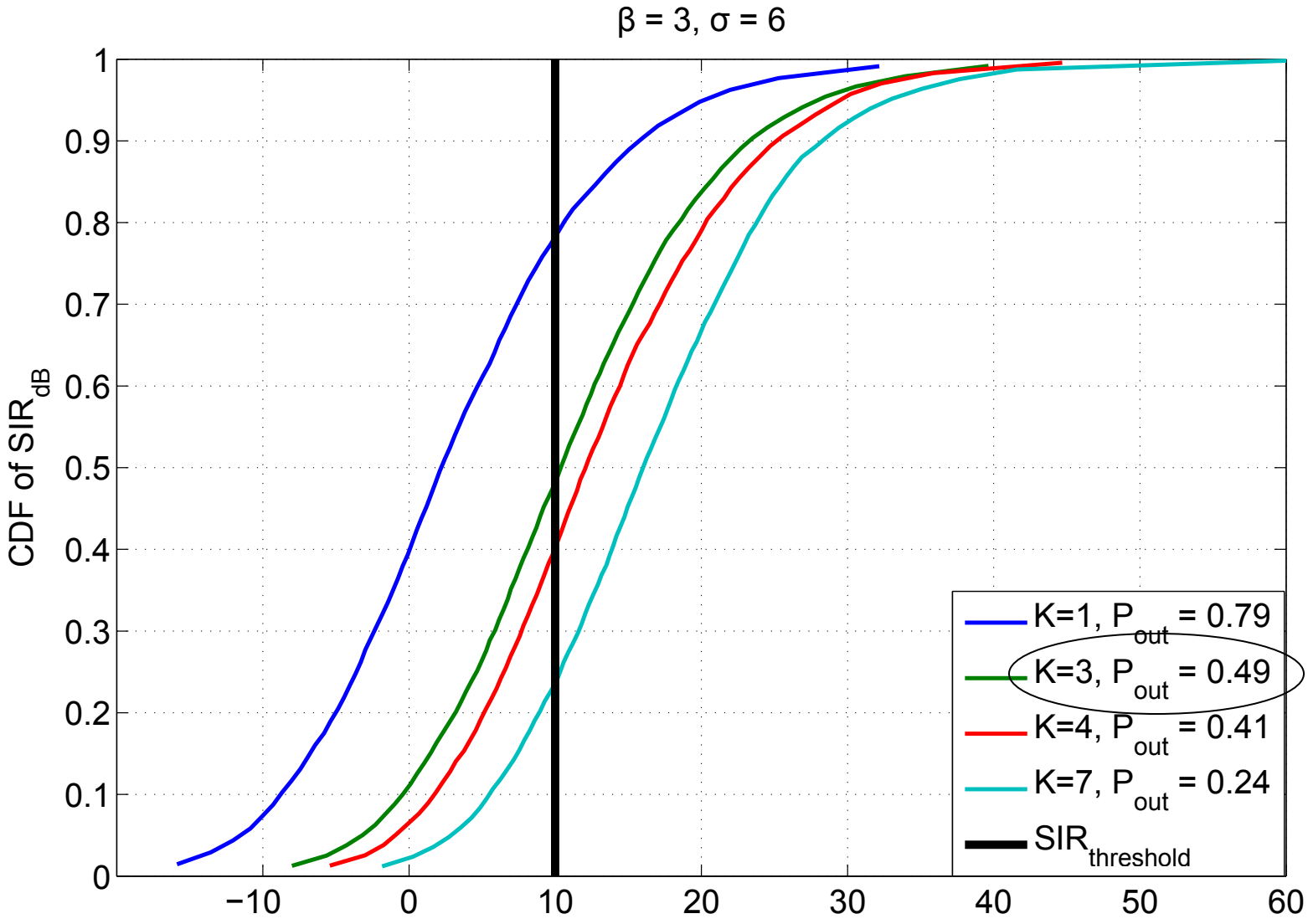
Reuse: Simulations



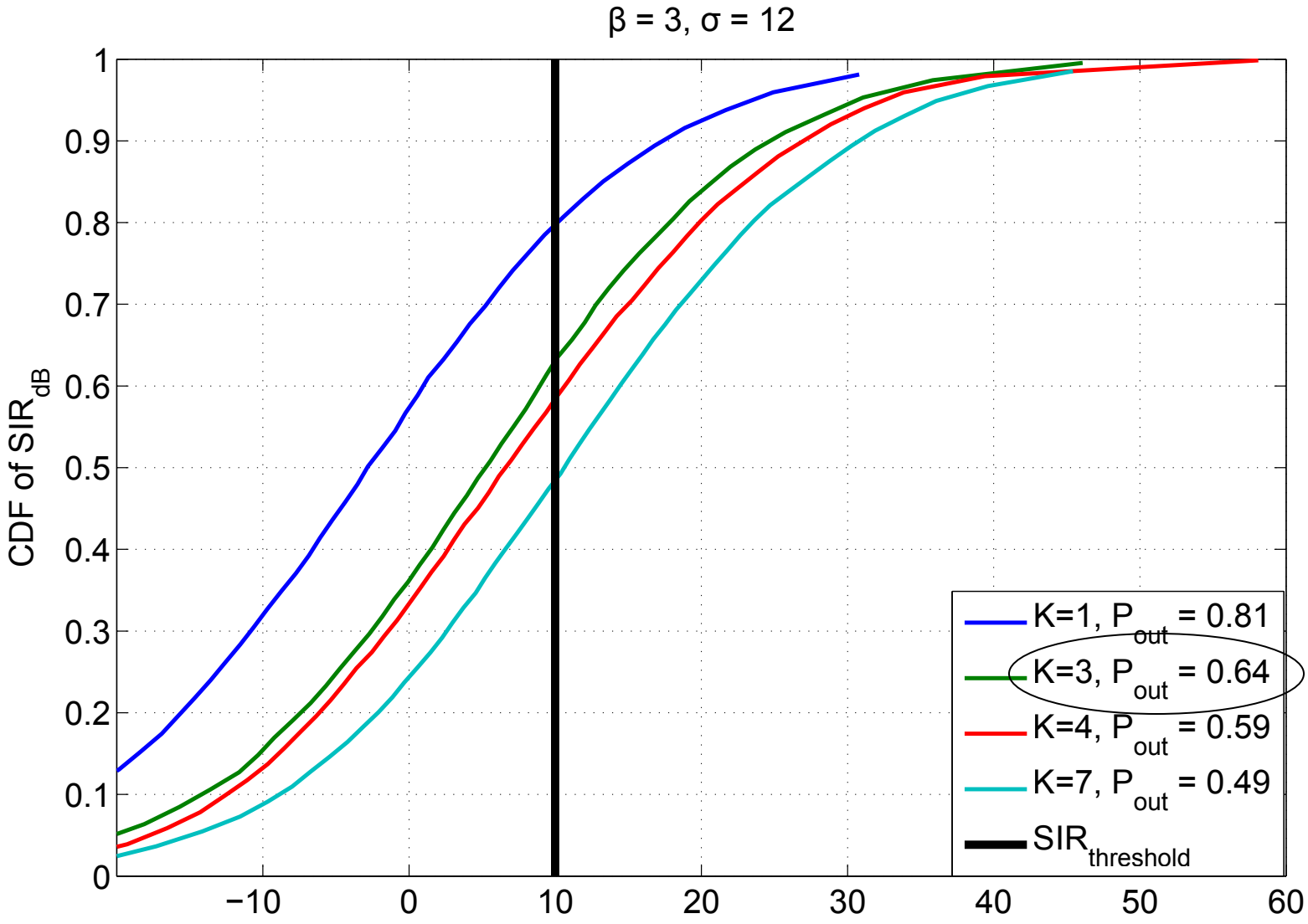
Reuse: Simulations



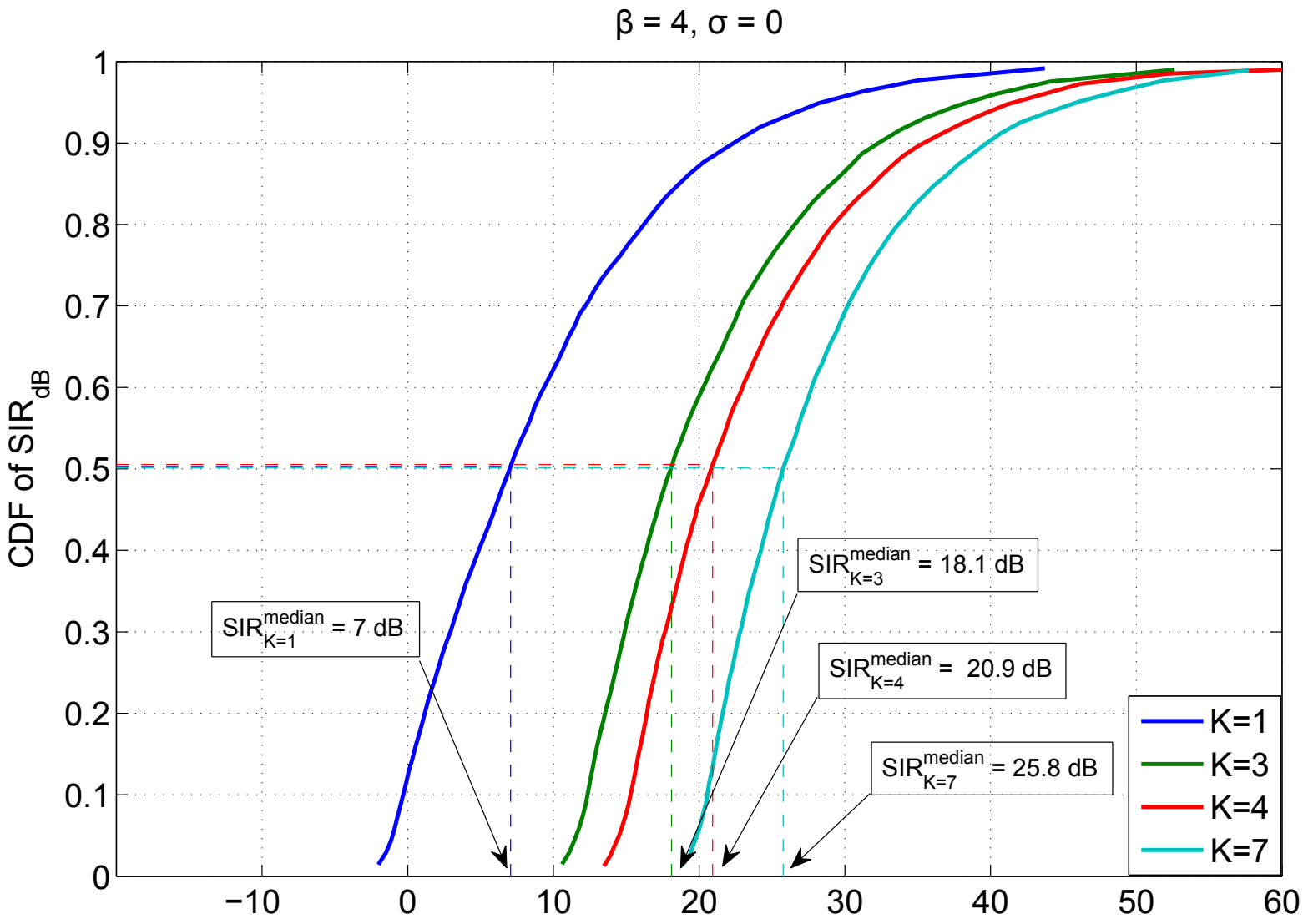
Reuse: Simulations



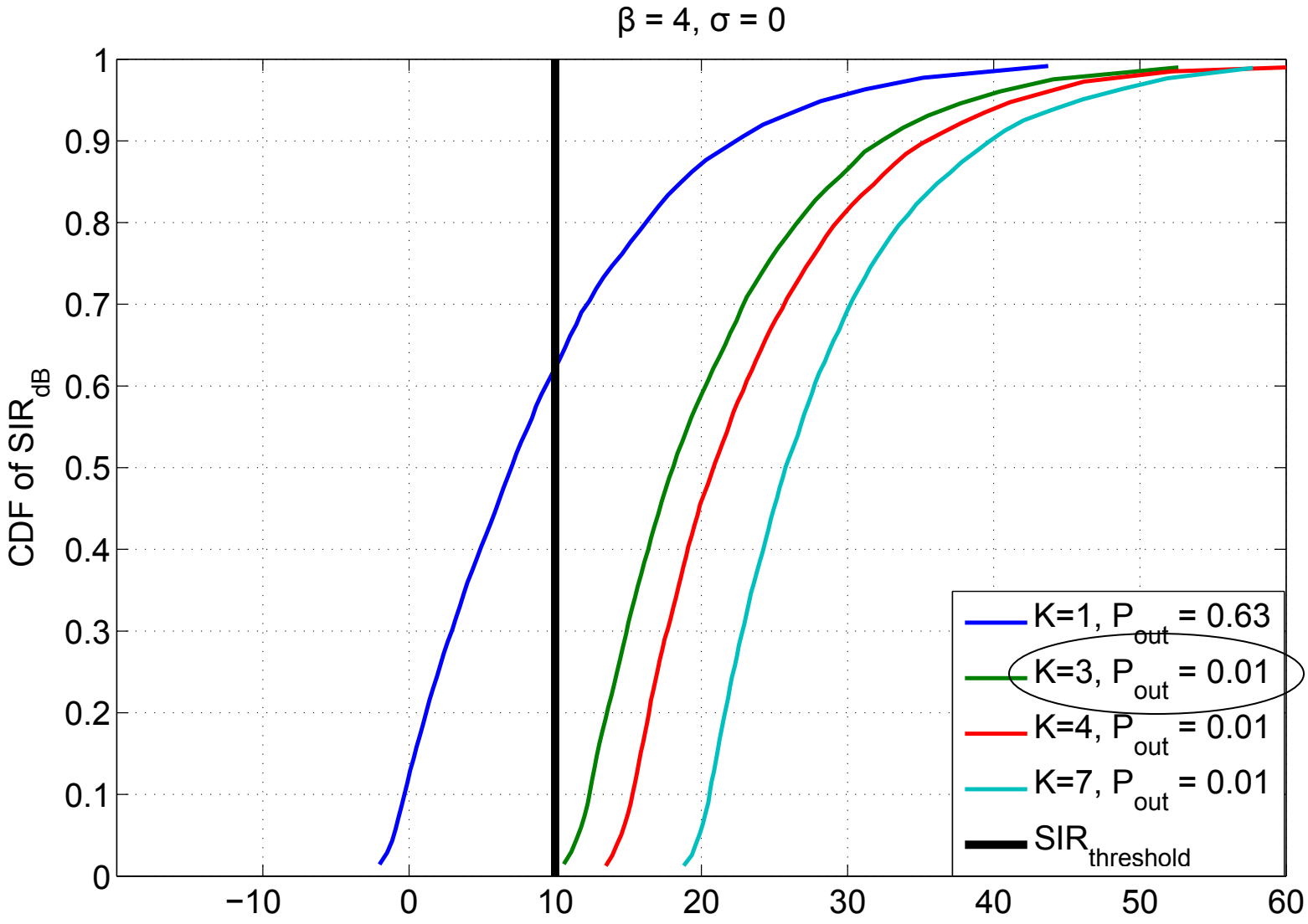
Reuse: Simulations



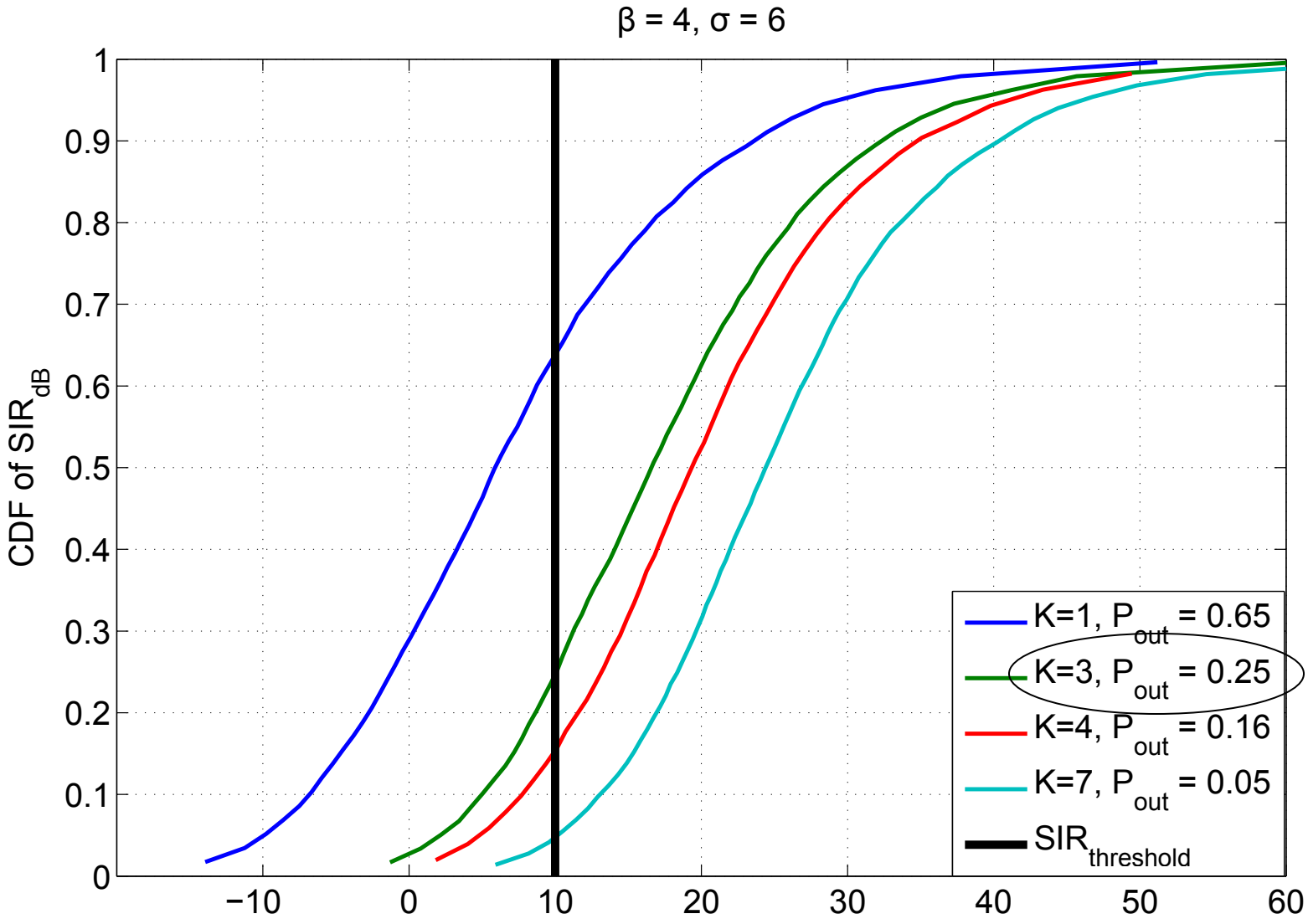
Reuse: Simulations



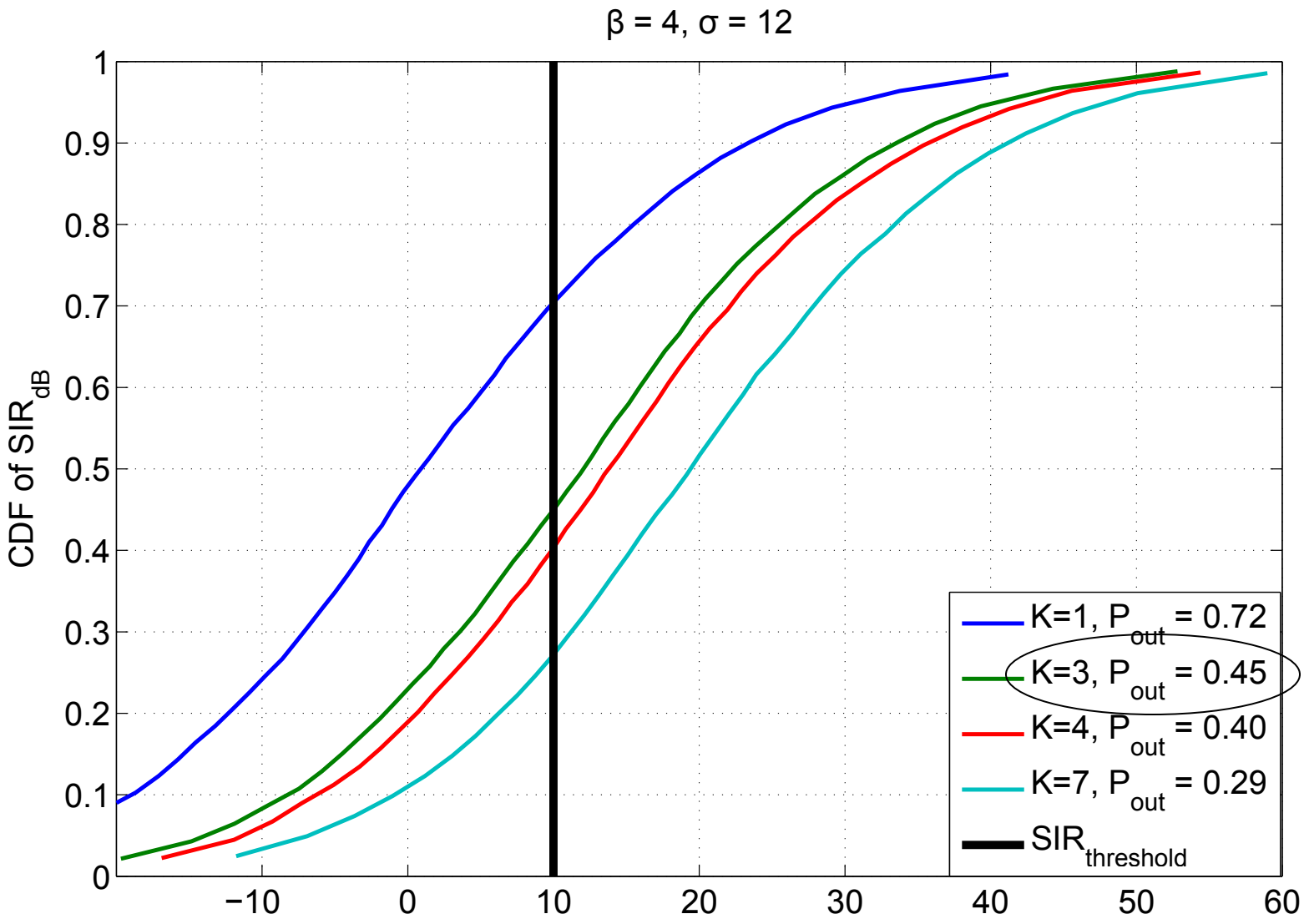
Reuse: Simulations



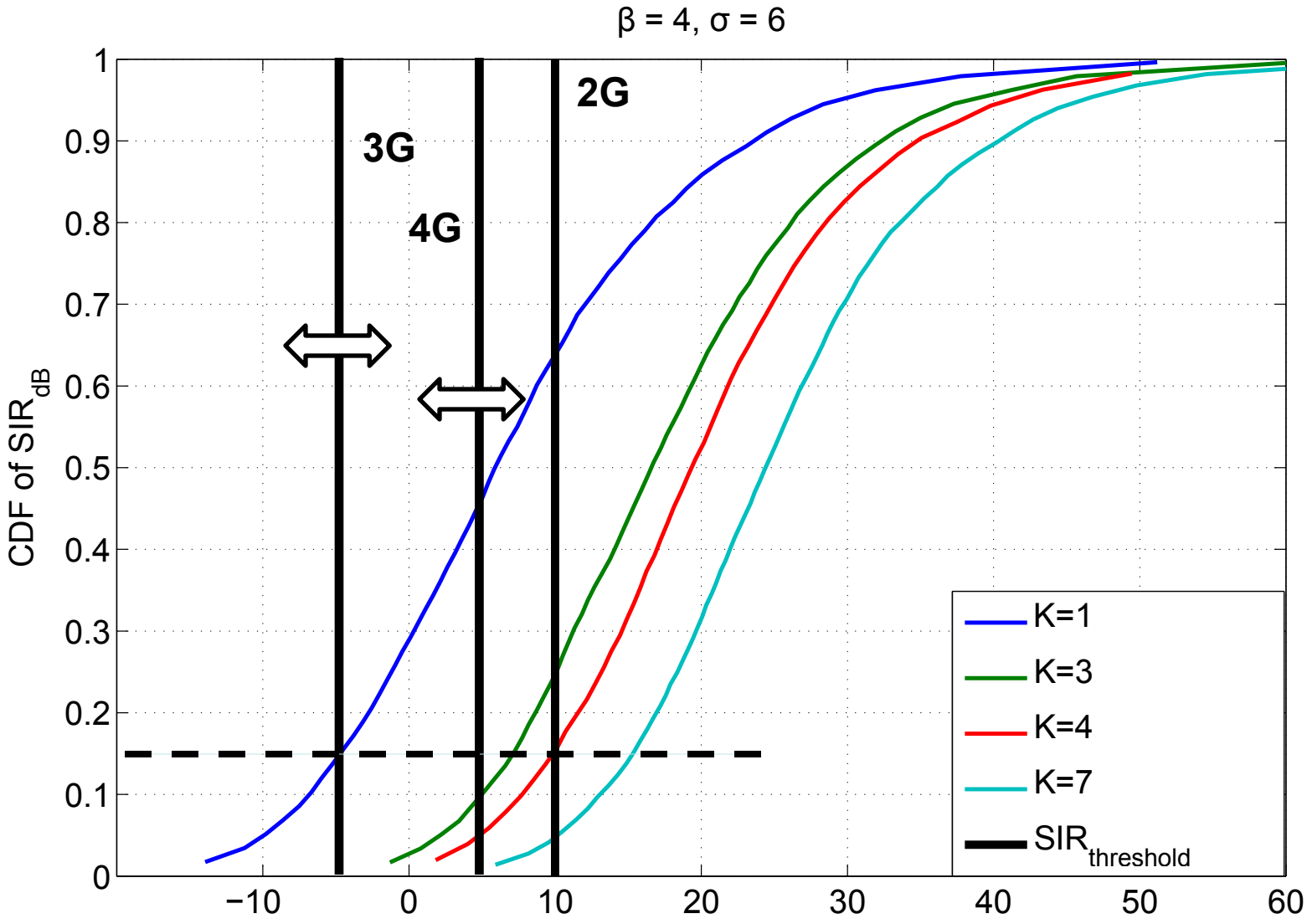
Reuse: Simulations



Reuse: Simulations



Reuse: Simulations. From 2G to 4G



3. Network Spectrum Efficiency

Network Spectrum Efficiency

N	number of RUs available to the system
K	number of cells per cluster
n = N / K	number of RUs per cell available
Bt	total bandwidth available [MHz]
Bc = Bt / N	equivalent bandwidth per RU [MHz]
A	cell area [sqKm]
Rb	minimum bit rate
E	carried traffic within cell [Erlang/cell]
e = E / n	carried traffic within cell per RU [Erlang/cell/RU]

Link Spectrum Efficiency $\eta_{link} = Rb / Bc$ [bit / sec / Hz]

It measures the efficiency of modulation and coding schemes.

How to measure Network Spectrum Efficiency?

It measures the efficiency of spectrum usage with reuse.
Many definitions available. None really general.

Network Spectrum Efficiency

Network Spectrum Efficiency **[Hatfield, 1982]**

$\eta = E / Bt\ A$ **[Erl/MHz/sqKm]**

(mainly valid for voice)

$\eta = E / Bt\ A = E / Bc\ N\ A = \eta_{link}\ E / Rb\ n\ K\ A = \eta_{link}\ e / Rb\ K\ A$

Network spectrum efficiency seems to be augmented if link spectrum efficiency is larger.

On the other hand, if the latter is increased at the expense of larger sensitivity to interference effects, K will be increased to fulfil QoS requirements.

So, this definition captures the tradeoff between QoS and efficiency, provided we succeed in expressing e as a function of K.

Network Spectrum Efficiency

N	number of RUs available to the system
K	number of cells per cluster
n = N / K	number of RUs per cell available
Bt	total bandwidth available [MHz]
Bc = Bt / N	bandwidth per RU [MHz]
A	cell area [sqKm]
Rb	minimum bit rate
S	**carried throughput within cell [bit/s/cell]**
s = S / n	**carried throughput within cell per RU [bit/s/cell/RU]**

Network Spectrum Efficiency (Hatfield modified)

$$\eta = S / Bt\ A \quad \text{[bit/s/MHz/sqKm]}$$

$$= s / Bc\ K\ A$$

Network Spectrum Efficiency

The maximum carried throughput per RU can be estimated through the concept of channel capacity (Shannon):

$$s = Bc \log_2 (1 + SNR)$$

Assuming SNR = C/I (not true, but good approximation for many interferers):

$$\eta = s / Bc\, K\, A = \log_2 (1 + C/I) / K\, A$$

$$= \log_2 (1 + (3\, K)^{\beta/2} / Nint) / K\, A$$

Network spectrum efficiency is maximum for K = 1.

However $C/I = (3\, K)^{\beta/2}$ does not account for very unlucky conditions (edge users with close interferers, and affected by deep shadowing).

Cluster size dimensioning requires consideration of shadowing.

4. Cluster Size Dimensioning

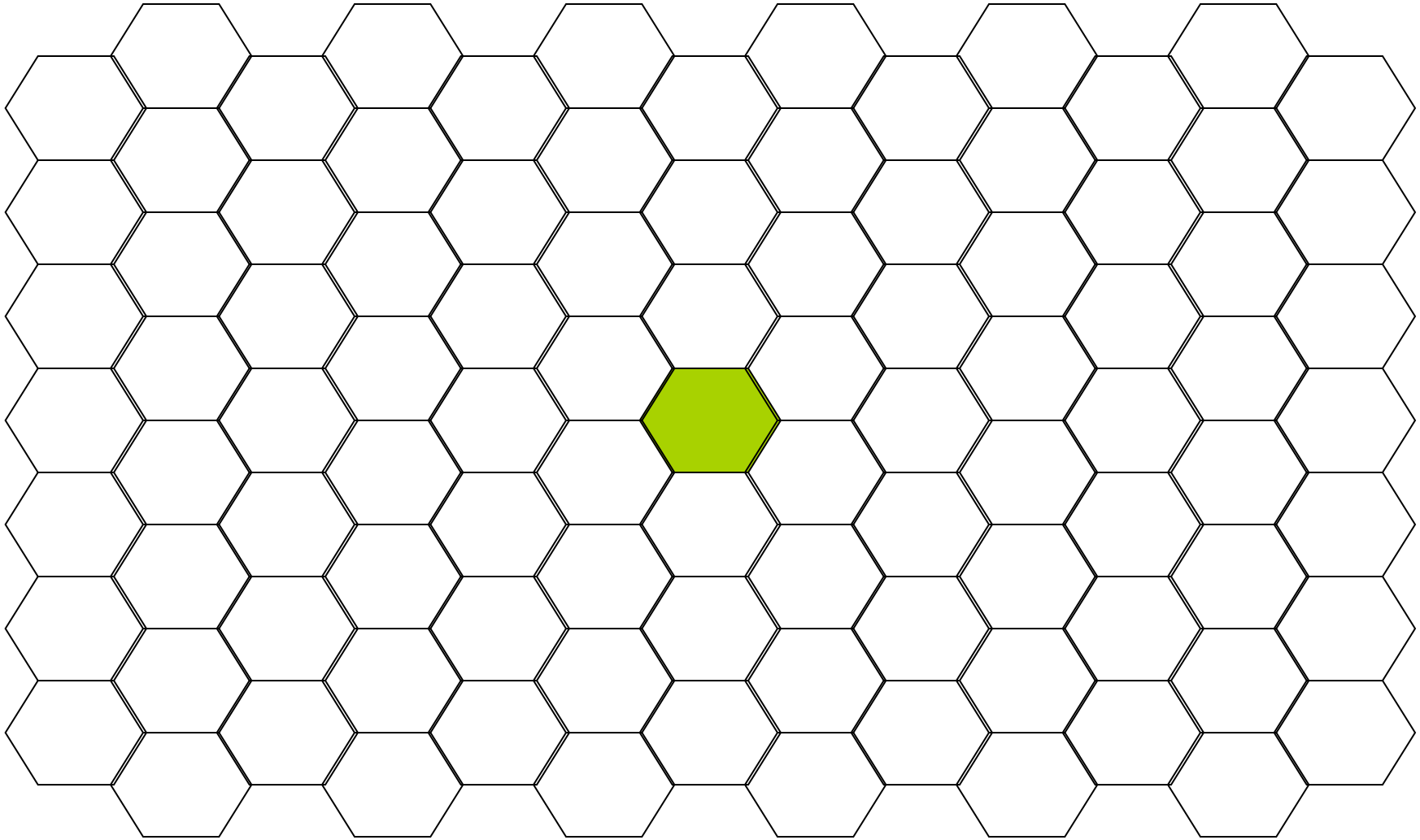
Cluster Size Dimensioning

Shadowing matters!

Math. derivation

5. Exercises

Homework



Homework

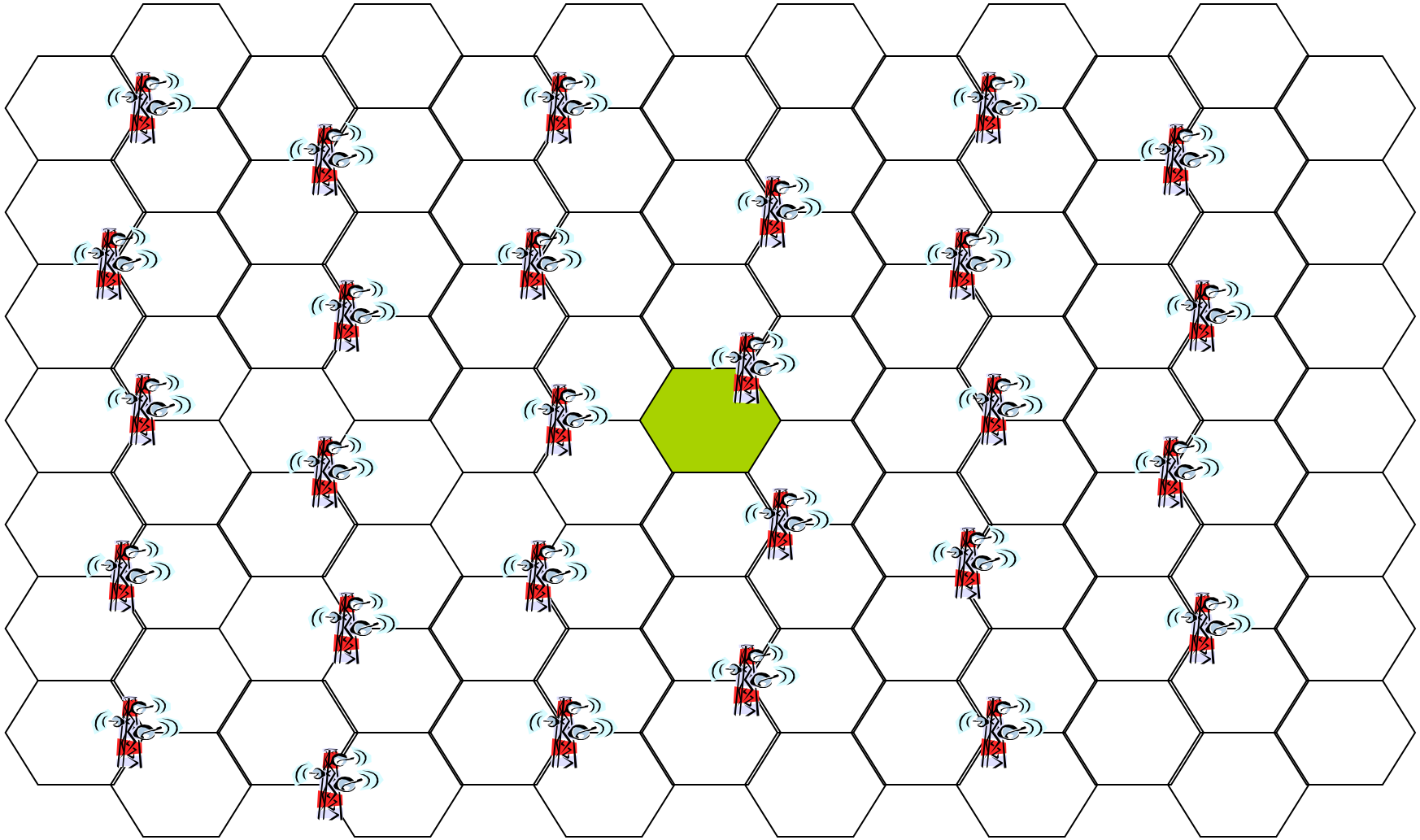
Assuming tri-sectorial antennas and K = 4, find the interfering cells of the central reference cell for uplink and downlink. The system uses FDD.

Are the two sets identical?

What does it change if the system uses asynchronous TDD?

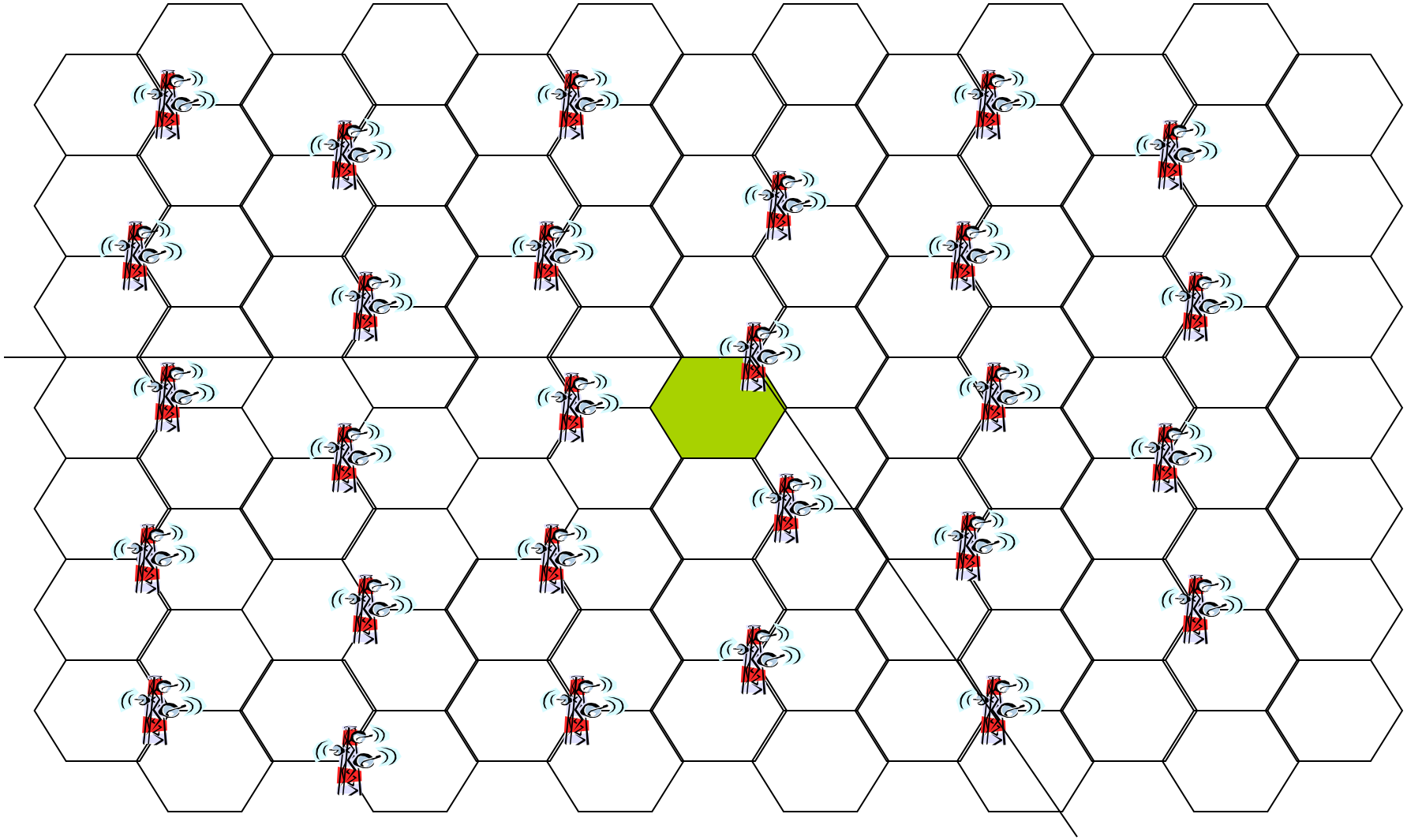
Homework

FDD



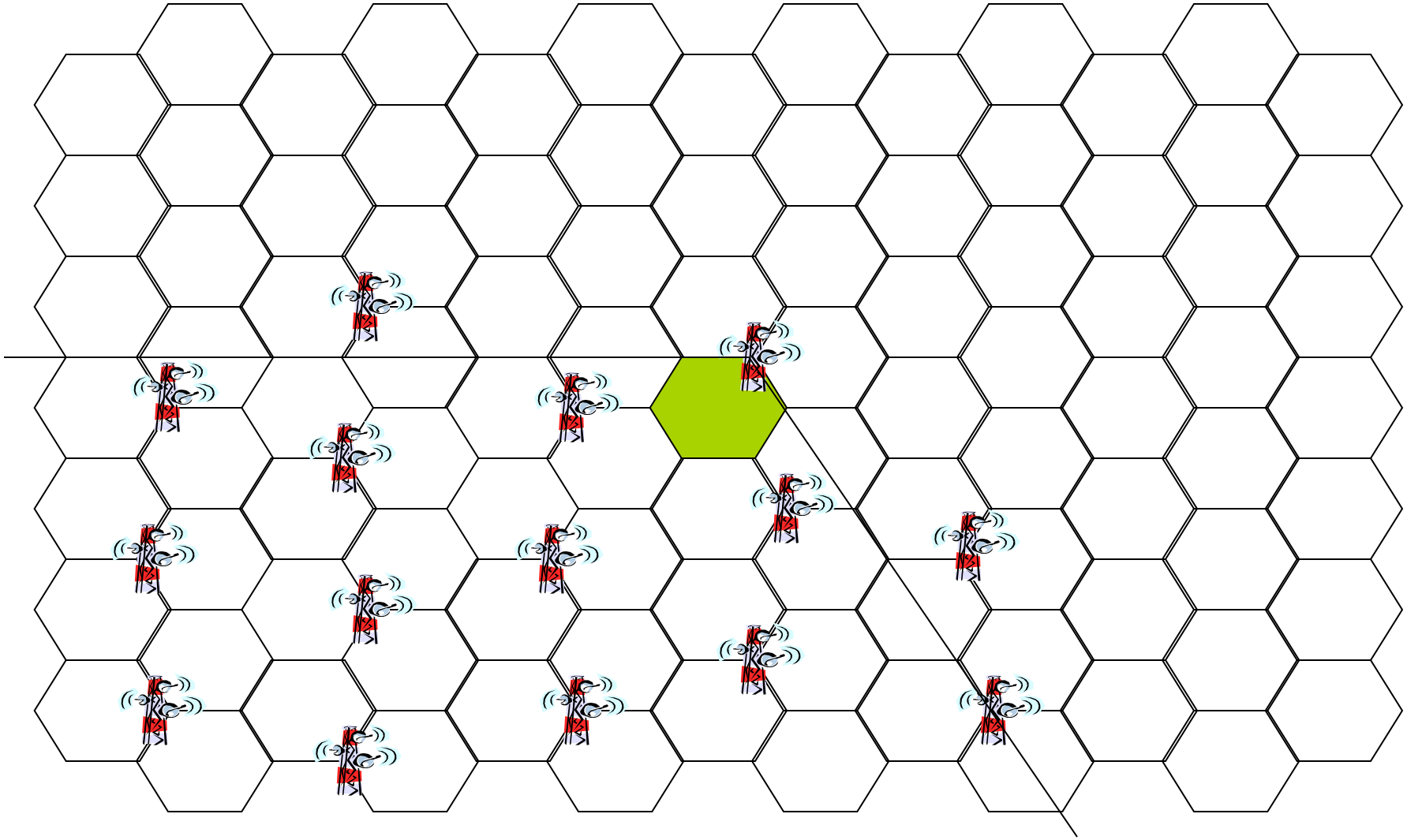
Homework

FDD



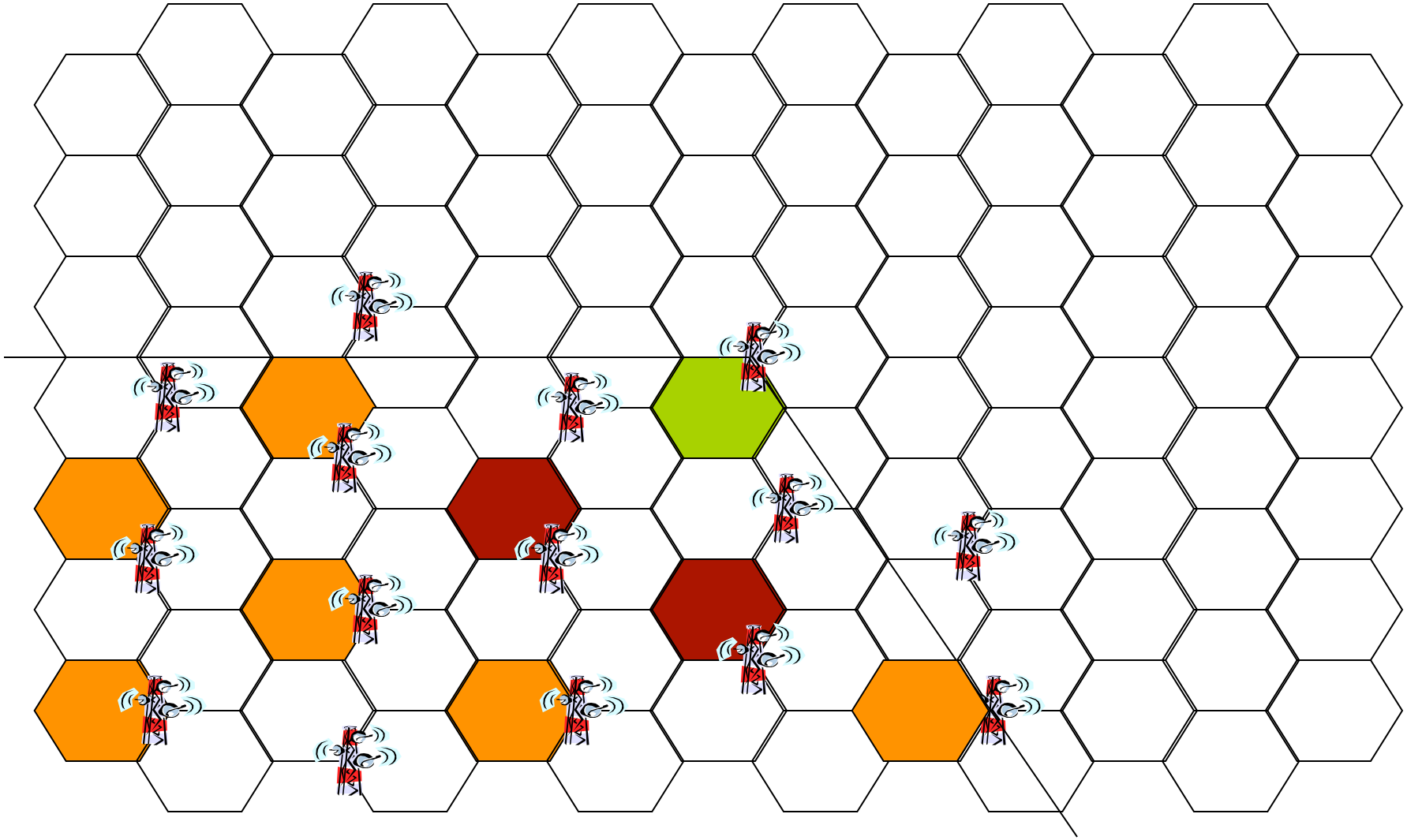
Homework

FDD: uplink



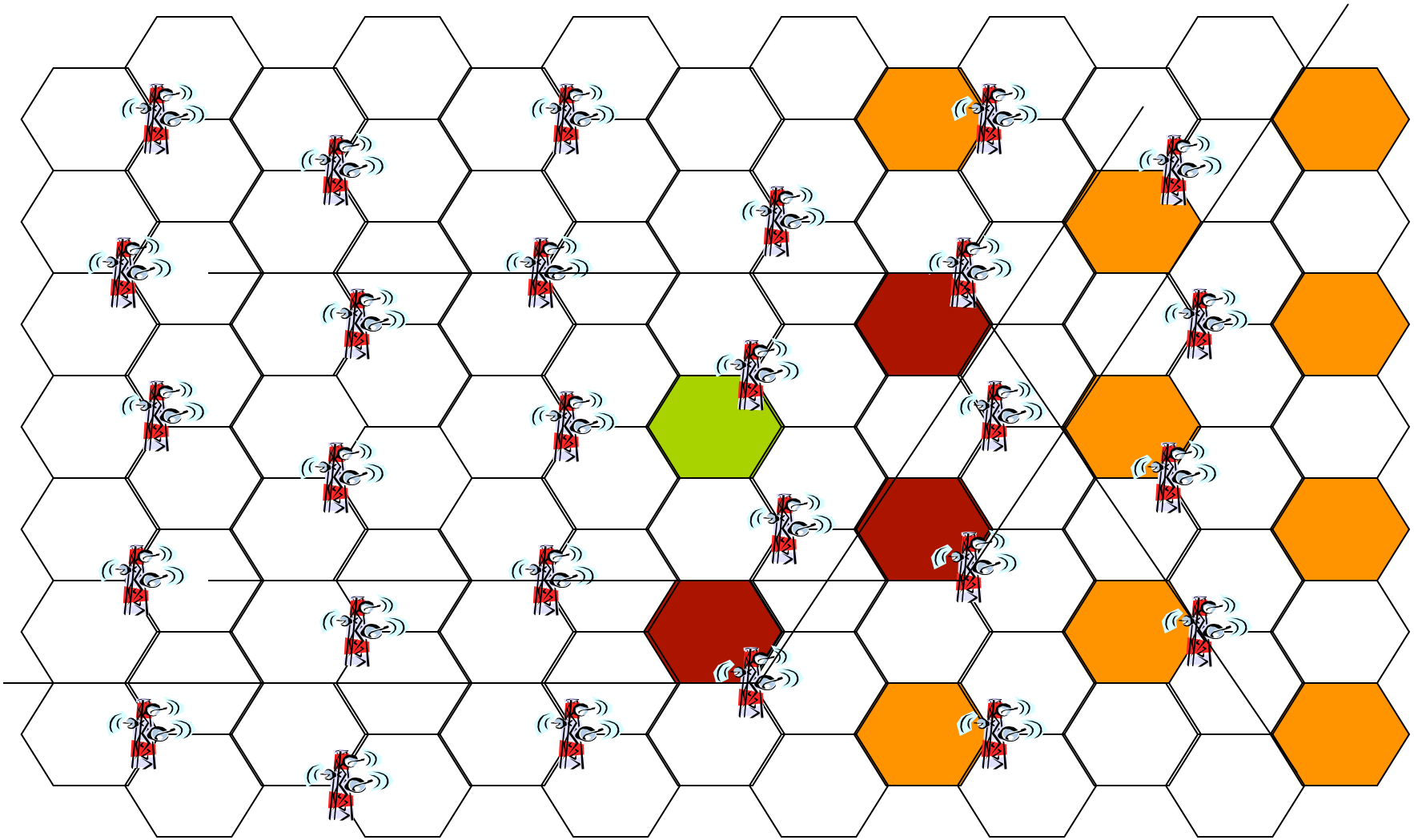
Homework

FDD: uplink



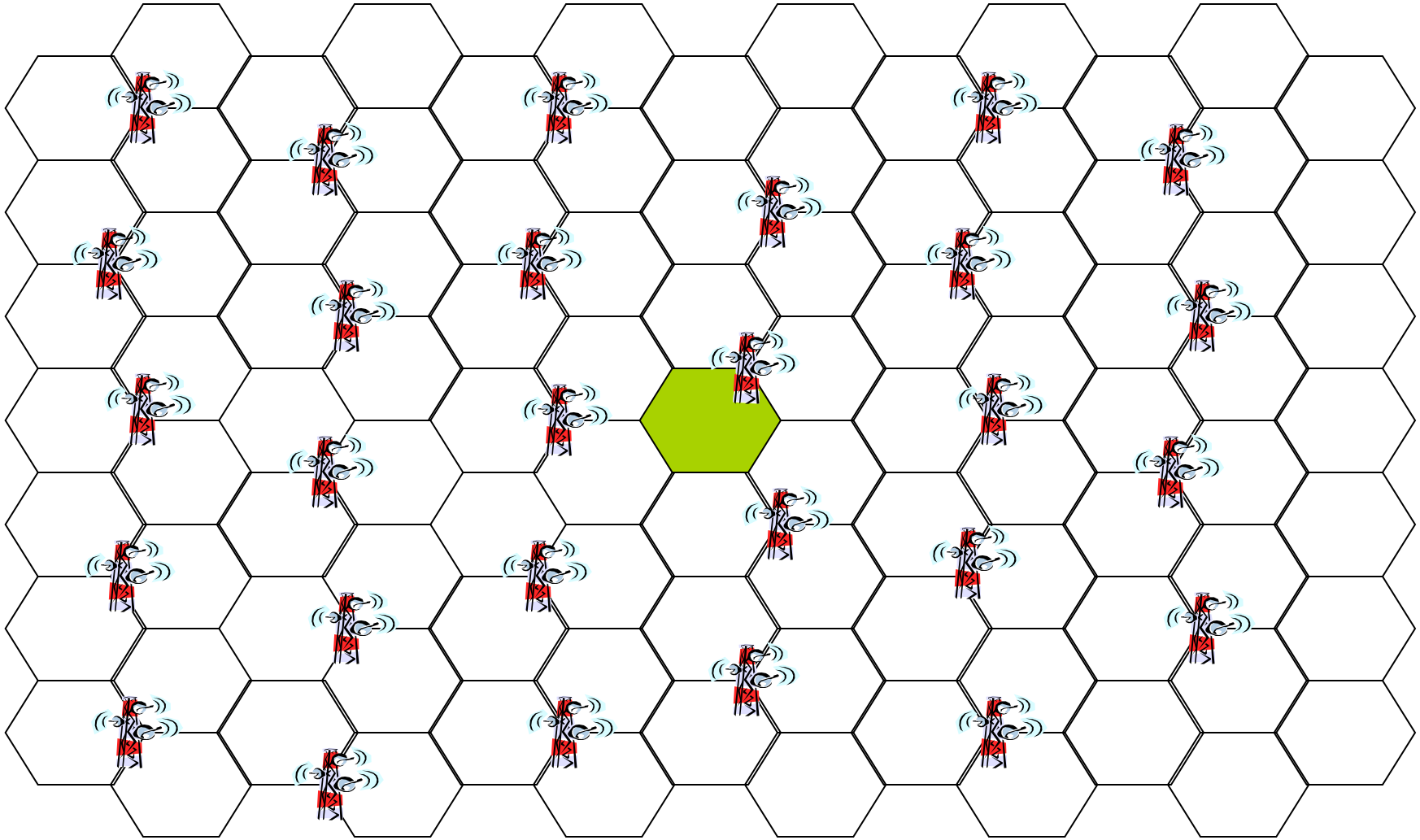
Homework

FDD: downlink



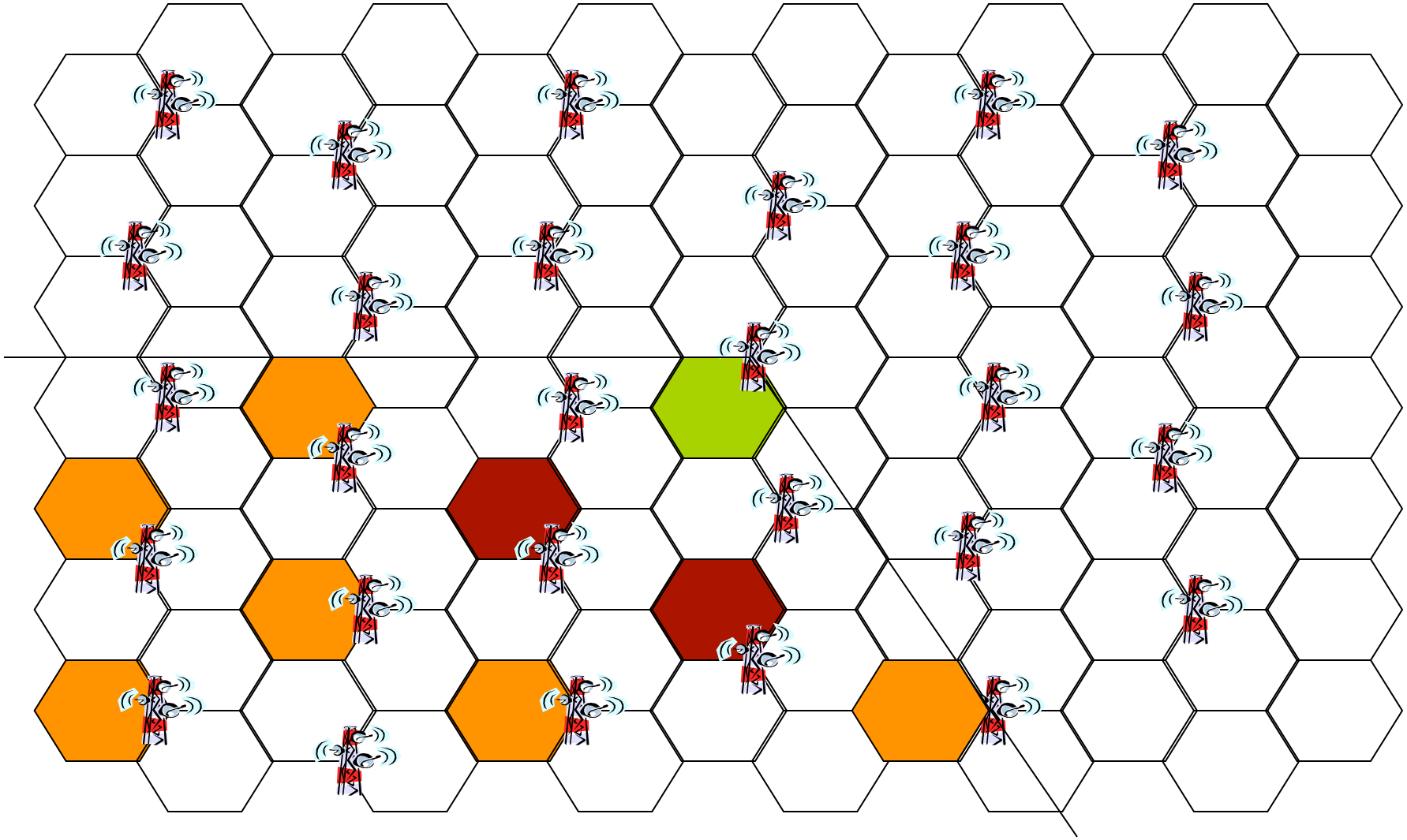
Homework

TDD



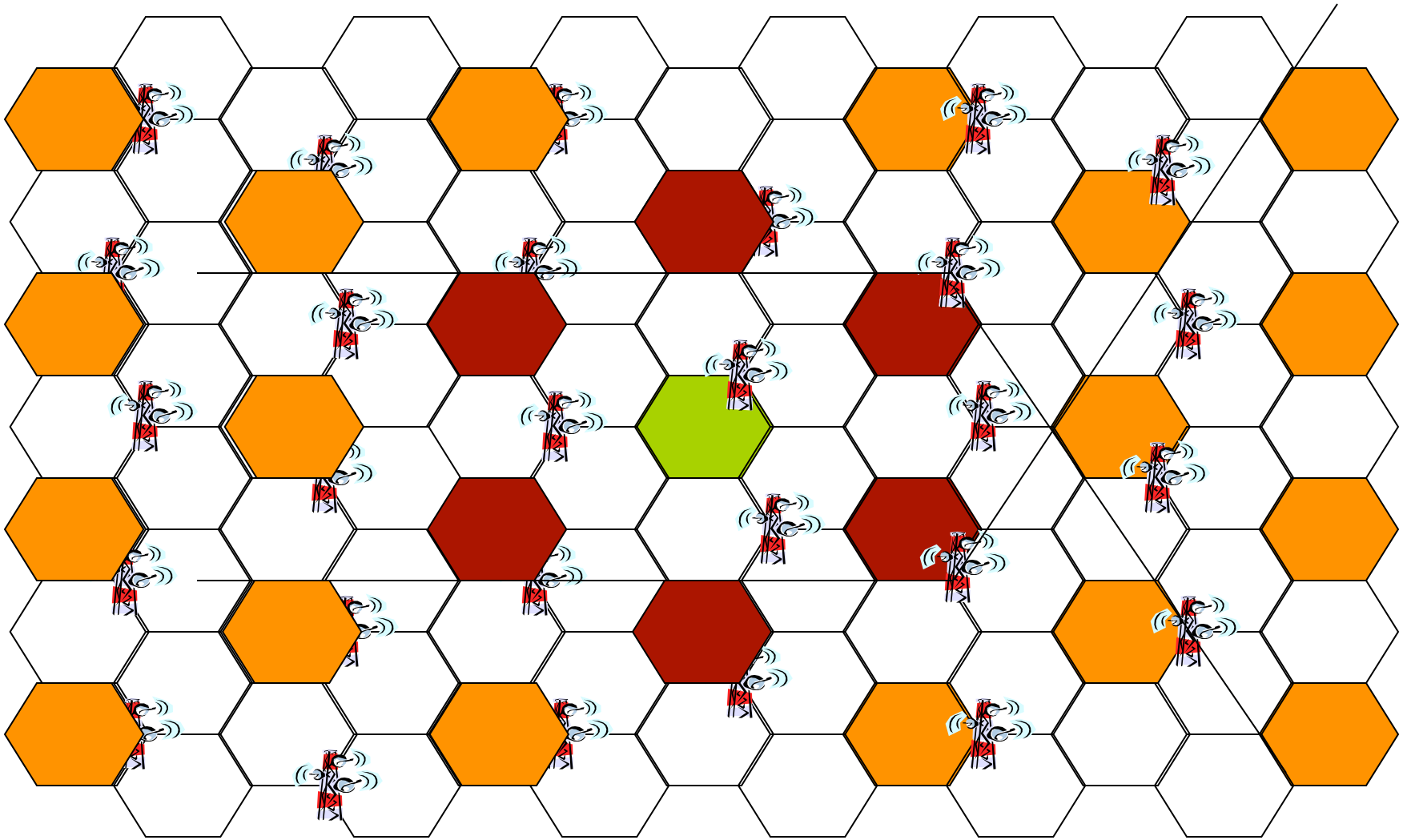
Homework

TDD: uplink



Homework

TDD: downlink



Homework

Assuming tri-sectorial antennas and K = 3, find the interfering cells of the central reference cell for uplink and downlink. The system uses FDD.

Are the two set identical?

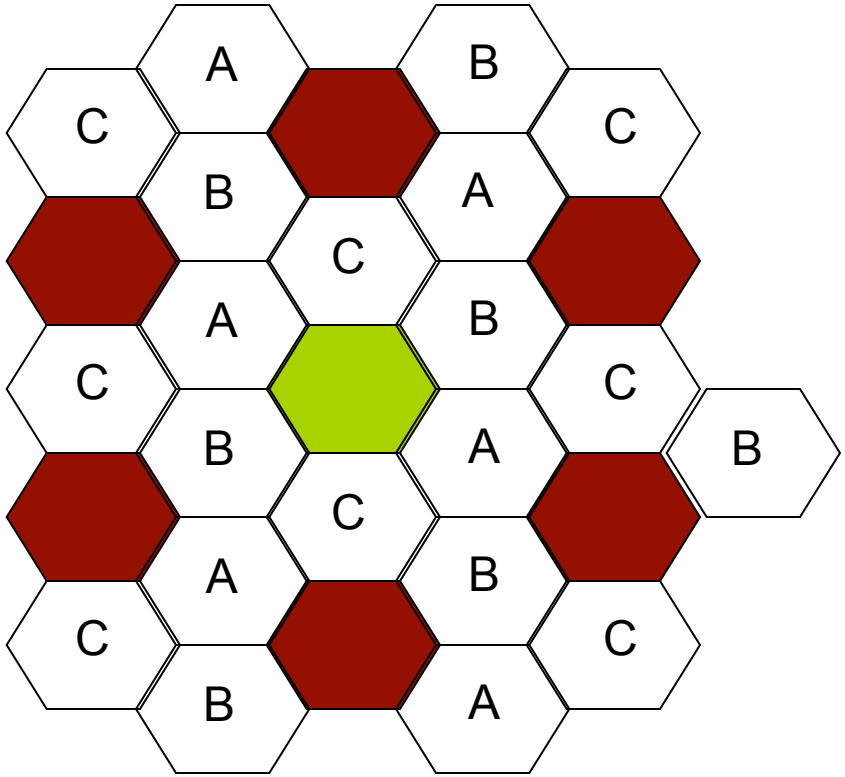
Is the value of Nint equal to 2 in all cases?

Homework

A service area is covered through 28 cells.
The radio resource set is composed of 120 RUs.
The minimum required CIR is 7 dB.
Antennas are trisectorial.

Assuming free space radio propagation, and uniform distribution of users, assign the RUs to the cells.

What is the maximum number of users that can be served in the service area (network capacity)?

Homework

C/I > 7 dB
→ K = 4
→ 7 clusters
→ n = 30
→ Max n. users = 30*28